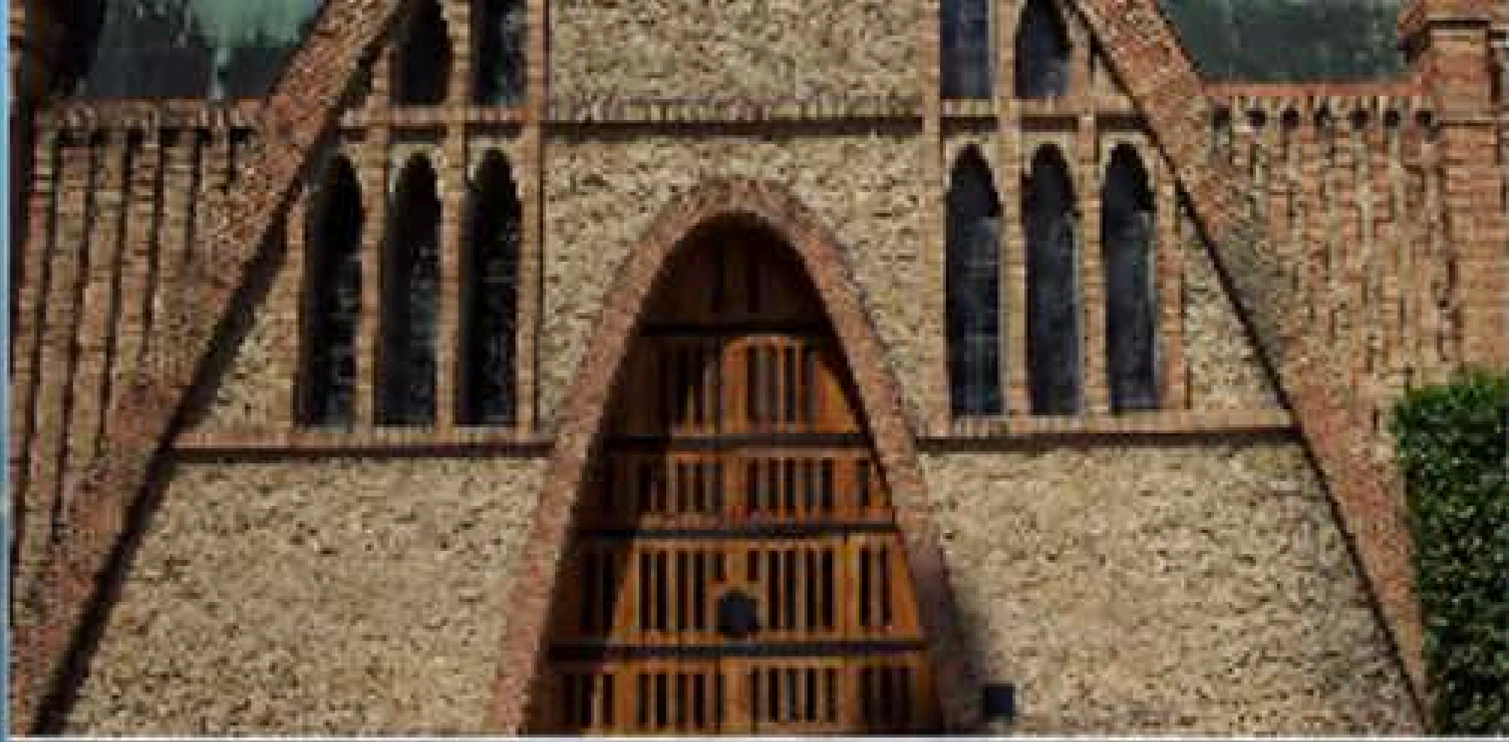
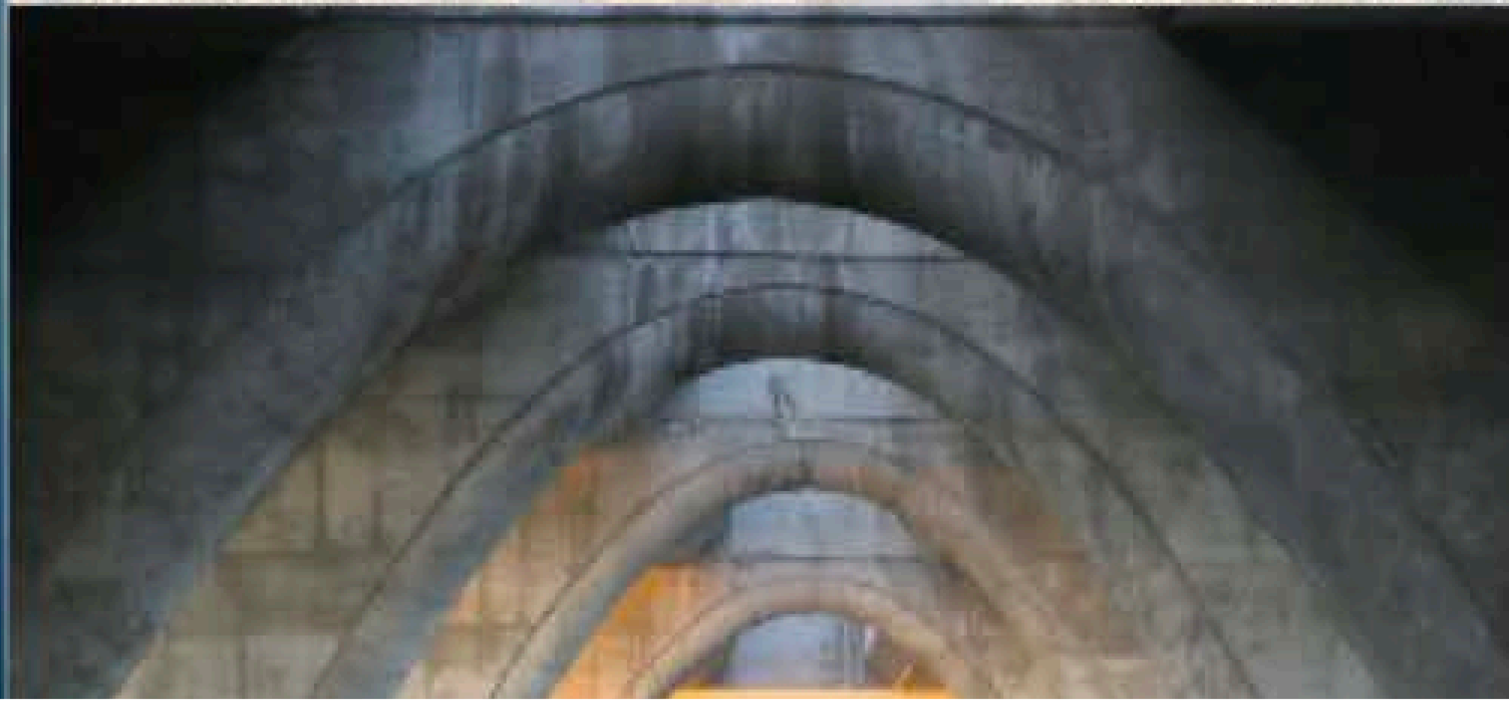
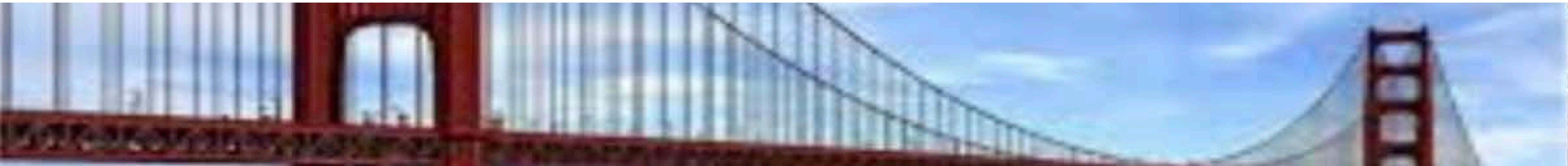
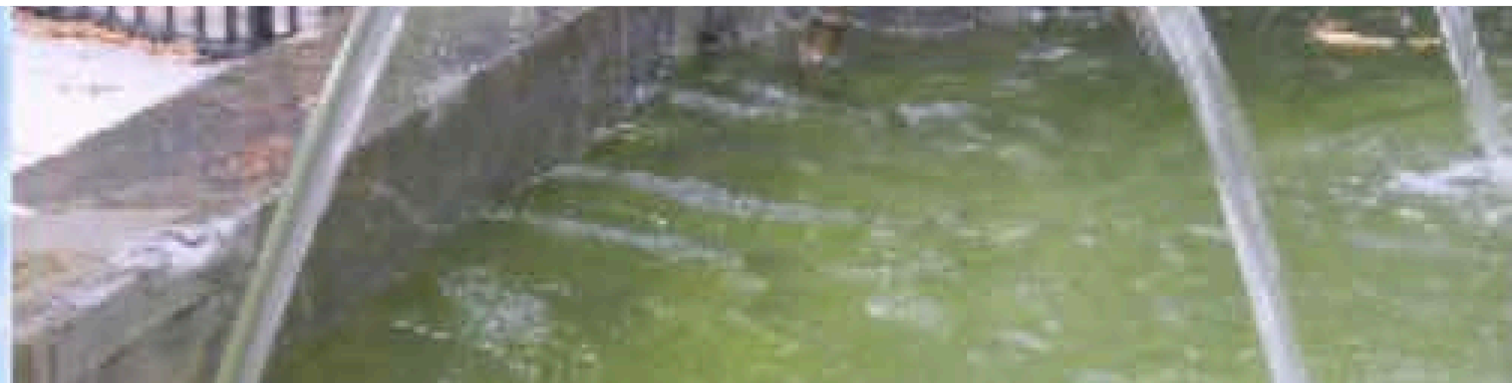




QUADRATIC FUNCTIONS

Chapter 2 Form 4

Quadratic Equation

Prior Knowledge (Mathematics Form 4)

Quadratic Equation

General form
$ax^2 + bx + c = 0$
$a \neq 0$

$2x^2 + 3x - 9 = 0$

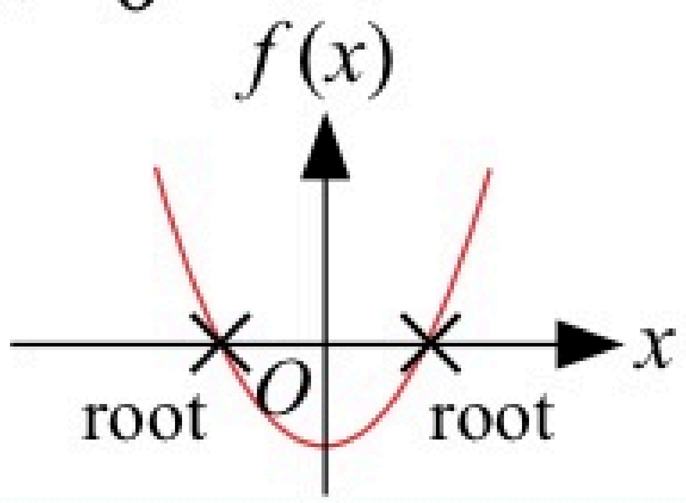
$(2x - 3)(x + 3) = 0$

$x = \frac{3}{2}$ or $x = -3$

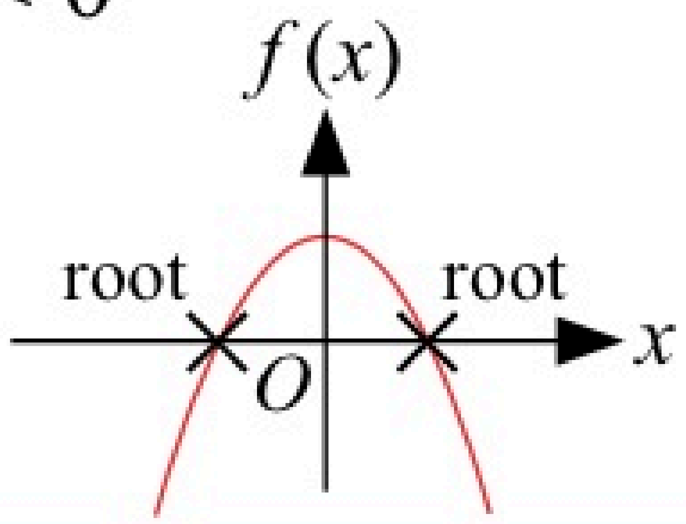
The roots of a quadratic equation are the values of the variable that satisfy the equation

The roots of a quadratic equation can be determined using
(a) factorisation method
(b) graphical method

$a > 0$



$a < 0$



Quadratic Functions

Additional Mathematics Form 4

Quadratic Equation

Solving Quadratic Equations by using the method of completing the square and quadratic formula

Forming Quadratic Equation From Given Roots

Type of Roots of Quadratic Equations

Quadratic Inequalities

Solving Quadratic Equations by using the method of completing the square and quadratic formula

Example 1

Solve the following quadratic equations by using completing the square method.

(*a*) $x^2+4x-7=0$

(*b*) $-3x^2+6x-1=0$

Solving Quadratic Equations by using method of completing the square

Solve the following quadratic equations by using completing the square method.

(a) $x^2+4x-7=0$

(b) $-3x^2+6x-1=0$

Solution 1:

(a) $x^2+4x-7=0$

Move the constant term to the right hand side of the equation

$$x^2+4x=7$$

Add the term $\left(\frac{\text{coefficient of } x}{2}\right)^2$ on the left and right hand side of the equation

$$x^2+4x+\left(\frac{4}{2}\right)^2=7+\left(\frac{4}{2}\right)^2$$

$$x^2+4x+2^2=7+2^2$$

$(x+a)^2 = x^2 + 2ax + a^2$

$$(x+2)^2=11$$

$$x+2=\pm\sqrt{11}$$

$$x=-5.317\ ,\ x=1.317$$

(b) $-3x^2+6x-1=0$

Divide both sides of the equation by –3 so that the coefficient of x^2 becomes 1

$$x^2-2x+\frac{1}{3}=0$$

$$x^2-2x=-\frac{1}{3}$$

Add $\left(\frac{-2}{2}\right)^2$ on both sides of the equation

$$x^2-2x+\left(\frac{-2}{2}\right)^2=\frac{-1}{3}+\left(-\frac{2}{2}\right)^2$$

$$x^2-2x+(-1)^2=-\frac{1}{3}+(-1)^2$$

$$(x-1)^2=\frac{2}{3}$$

$$x-1=\pm\sqrt{\frac{2}{3}}$$

$$x=1.816\ ,\ x=0.184$$

Solving Quadratic Equations by using the method of quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 2

Solve the following quadratics equations. Write your answer in three decimal places.

(a) $5x^2 - 7x - 11 = 0$

(b) $m(6m - 1) = 7m + 13$

Solving Quadratic Equations by using the method of quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Solution 2a:

$(a)\ 5x^2-7x-11=0$

$a=5,\quad b=-7,\quad c=-11$

Identify the values of *a*, *b* and *c* first

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(5)(-11)}}{2(5)}$$

Be aware of the sign (-). For example, b^2 is $(-7)^2$ instead of -7^2

$x=-0.940\ ,\quad x=2.340$

Use a calculator to get this value

Solve the following quadratics equations. Write your answer in three decimal places.

$(a)\ 5x^2-7x-11=0$

$(b)\ m(6m-1)=7m+13$

Solving Quadratic Equations by using the method of quadratic formula

Solution 2b:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Solve the following quadratics equations. Write your answer in three decimal places.

(a) $5x^2-7x-11=0$

(b) $m(6m-1)=7m+13$

$$m(6m-1)=7m+13$$

$$6m^2-m-7m-13=0$$

$$6m^2-8m-13=0$$

Form a general form of quadratic equation *ax*² + *bx* + *c* = 0

$$a=6,\ \ b=-8,\ \ c=-13$$

Identify the values of *a*, *b* and *c* first

$$m=\frac{-(-8)\pm\sqrt{(-8)^2-4(6)(-13)}}{2(6)}$$

$$m=2.283\ ,\quad m=-0.949$$

Forming Quadratic Equations From Given Roots

The quadratic equation $ax^2+bx+c=0$ can be written as

$$x^2+\frac{b}{a}x+\frac{c}{a}=0 \qquad \ldots(1)$$

If α and β are the roots of a quadratic equation, then

$$x=\alpha, \quad x=\beta$$

$$x-\alpha=0, \quad x-\beta=0$$

$$(x-\alpha)(x-\beta)=0$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0 \qquad \ldots(2)$$

x^2 - (SOR) x + (POR) = 0

Comparing (1) and (2),

$$-(\alpha+\beta)=\frac{b}{a} \quad \text{and} \quad \alpha\beta=\frac{c}{a}$$

$$\alpha+\beta=-\frac{b}{a}$$

In general, this comparison can be formulated as follows:

Sum of roots, SOR $=\alpha+\beta=-\frac{b}{a}$

Product of roots, POR $=\alpha\beta=\frac{c}{a}$

Example 3

The quadratic equation $x^2 + (p - 5)x + 2q = 0$ has roots of -3 and 6. Find the value of p and q.

$$x^2 - [-(p-5)]x + 2q = 0$$

Solution 3 :

x^2 - (SOR) x + (POR) = 0

Sum of root $= -(p - 5)$

Sum of root = $-3 + 6 = 3$

$$5 - p = 3$$

$$p = 5 - 3$$

$$p = 2$$

Product of root $= 2q$

Product of root = $-3 \times 6 = -18$

$$2q = -18$$

$$q = -9$$

$x = -3, x = 6$

$x + 3 = 0, x - 6 = 0$

$(x + 3)(x - 6) = 0$

$x^2 + 3x - 18 = 0$

$x^2 - [-(p - 5)]x + 2q = 0$

$5 - p = 3$ $\quad$ $2q = -18$

$p = 5 - 3$ $\quad$ $q = -9$

$p = 2$

Example 4

Given the quadratic equation $3x^2 + px - 8 = 0$, where p is a constant. Find the value of p if

(a) one of the roots of the equation is -2,

(b) the sum of roots of the equation is $\frac{1}{3}$.

Solution 4:

x^2 - (SOR) x + (POR) = 0

(a)

$$3x^2 + px - 8 = 0$$
$$3(-2)^2 + p(-2) - 8 = 0$$
$$12 - 2p - 8 = 0$$
$$4 - 2p = 0$$
$$2p = 4$$
$$p = 2$$

(b)

$$3x^2 + px - 8 = 0$$
$$\frac{3x^2}{3} + \frac{px}{3} - \frac{8}{3} = 0$$
$$x^2 + \frac{p}{3}x - \frac{8}{3} = 0$$
$$\text{SOR} = -\frac{p}{3}$$
$$\frac{1}{3} = -\frac{p}{3}$$
$$p = -1$$

Solving Quadratic Inequalities

Graph Sketching

Number Line

Table

Example 5

Find the range of values of x for the quadratic inequality $(2x - 1)(x + 4) \geq x + 4$ by using

(a) graph sketching method

(b) number line method

(c) table method

Solution 5(a) :

Graph sketching method

$$(2x-1)(x+4) \geq x+4$$

$$2x^2+7x-4 \geq x+4$$

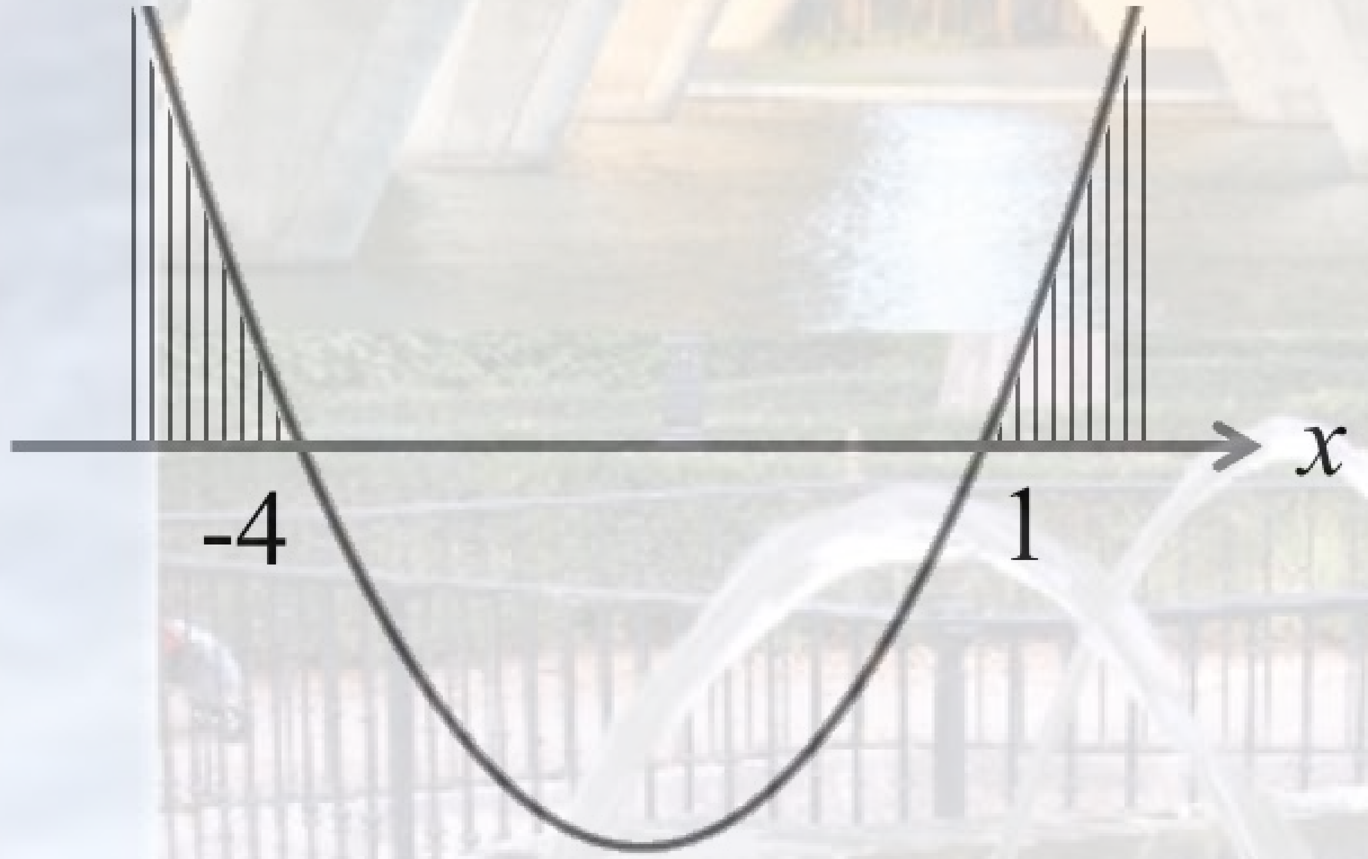
$$2x^2+6x-8 \geq 0$$

$$x^2+3x-4 \geq 0$$

$$(x+4)(x-1) \geq 0$$

$$x+4=0 \qquad x-1=0$$

$$x=-4 \qquad x=1$$



$x \leq -4 \,,\; x \geq 1$

Solving Quadratic Inequalities

Solution 5(b):

Number line method

Test point -5:
$x = -5$

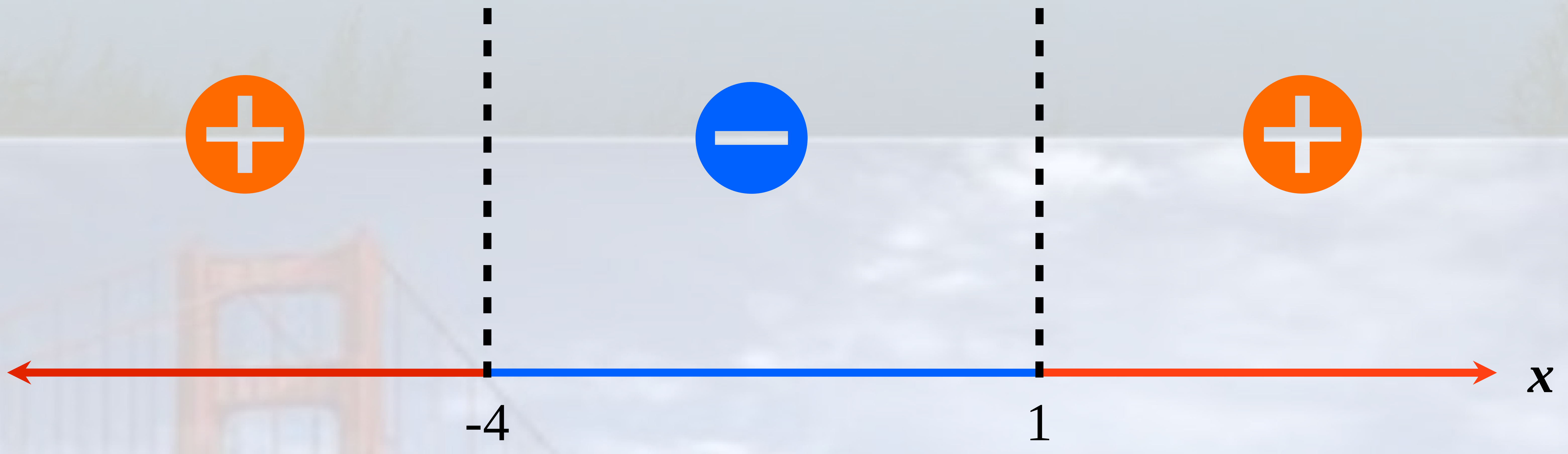
$(-5 + 4)(-5 - 1) \geq 0$
$(-1)(-6) \geq 0$

Test point 0:
$x = 0$

$(0 + 4)(0 - 1) \leq 0$
$(4)(-1) \leq 0$

Test point 2:
$x = 2$

$(2 + 4)(2 - 1) \geq 0$
$(6)(1) \geq 0$



$x \leq -4$ | $-4 \leq x \leq 1$ | $x \geq 1$

Since $(x + 4)(x - 1) \geq 0$, then the range of values of x is determined on the positive part of the number line.

Hence, the range of values of x is $x \leq -4$, $x \geq 1$

Find the range of values of x for the quadratic inequality $(2x - 1)(x + 4) \geq x + 4$ by using

(a) graph sketching method

(b) number line method

(c) table method

$(x + 4)(x - 1) \geq 0$

Solving Quadratic Inequalities

Solution 5(c):

Table method

Find the range of values of x for the quadratic inequality $(2x - 1)(x + 4) \geq x + 4$ by using

(a) graph sketching method

(b) number line method

(c) table method

	Range values of x		
	$x \leq -4$	$-4 \leq x \leq 1$	$x \geq 1$
$(x + 4)$	-	+	+
$(x - 1)$	-	-	+
$(x + 4)(x - 1)$	+	-	+

$$(x + 4)(x - 1) \geq 0$$

Since $(x + 4)(x - 1) \geq 0$, then the range of values of x is determined on the positive part of the table.

Hence, the range of values of x is $x \leq -4$, $x \geq 1$

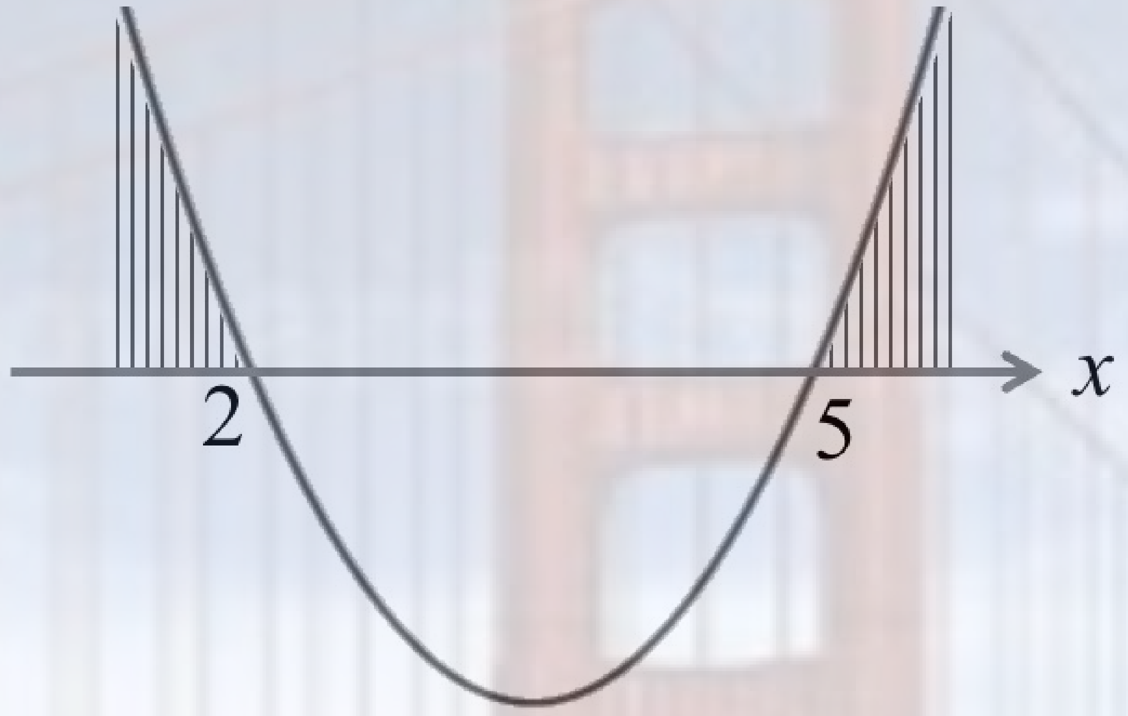
Example 6

Find the range of values of x for $x^2 - 7x + 10 > 0$ and $x^2 - 7x \le 0$. Hence, solve the inequality $-10 < x^2 - 7x \le 0$

Solution 6:

$$x^2 - 7x + 10 > 0$$

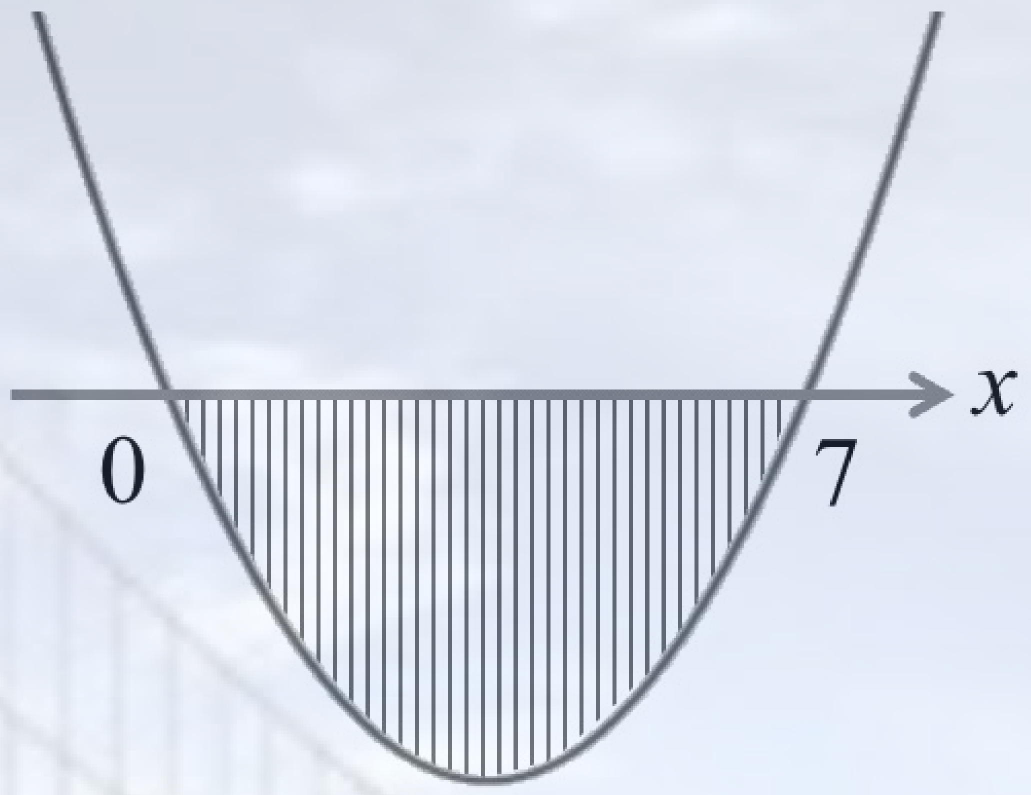
$$(x-2)(x-5) > 0$$



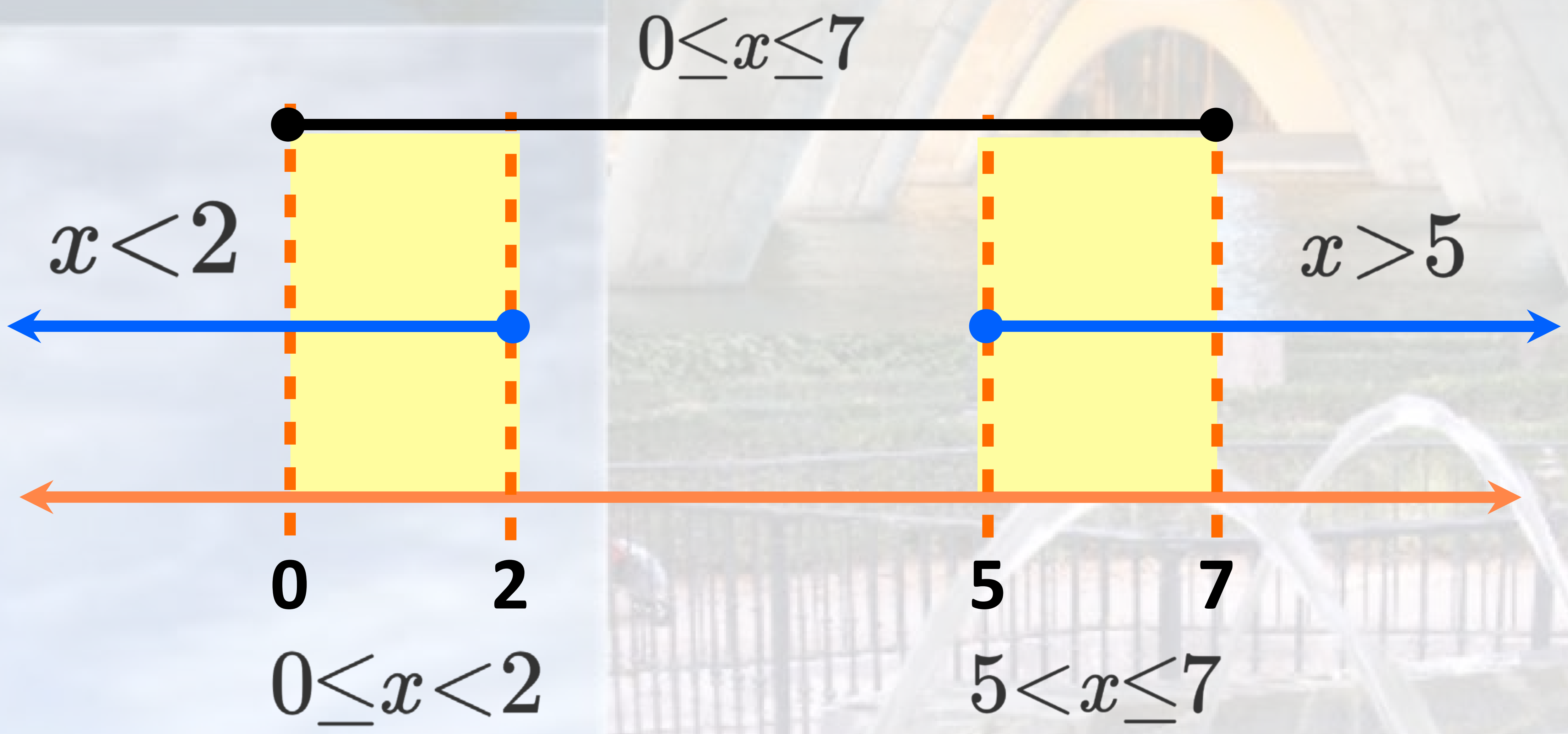
$$x < 2, \quad x > 5$$

$$x^2 - 7x \le 0$$

$$x(x-7) \le 0$$



$$0 \le x \le 7$$



$$0 \le x < 2, \quad 5 < x \le 7$$

Type of Roots of Quadratic Equations

Discriminant ($b^2 - 4ac$)

$y = x^2 + 5x + 4$

$y = 0, \quad x^2 + 5x + 4 = 0$

$a = 1, b = 5, c = 4$

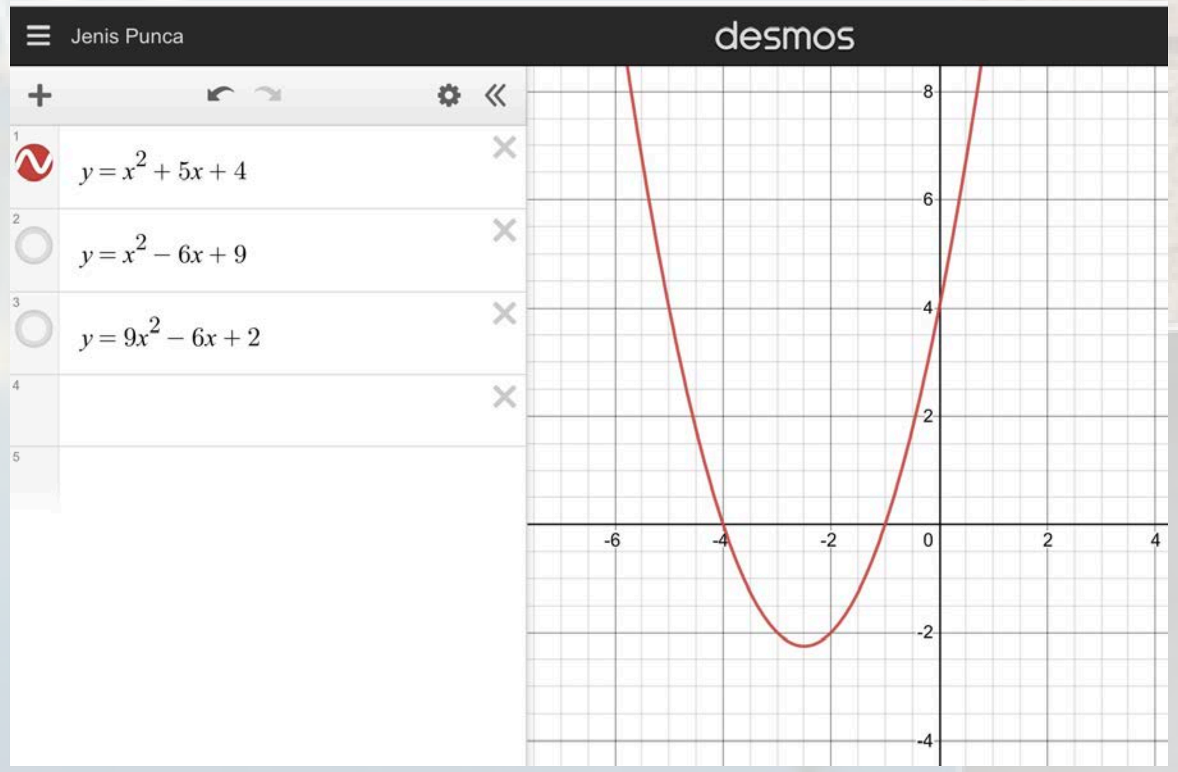
$b^2 - 4ac$

$= (5)^2 - 4(1)(4)$

$= 9 > 0$

$b^2 - 4ac > 0$

the equation has two different real roots



https://www.desmos.com/calculator/rhyqfobhpn

$y = x^2 - 6x + 9$

$y = 0, \quad x^2 - 6x + 9 = 0$

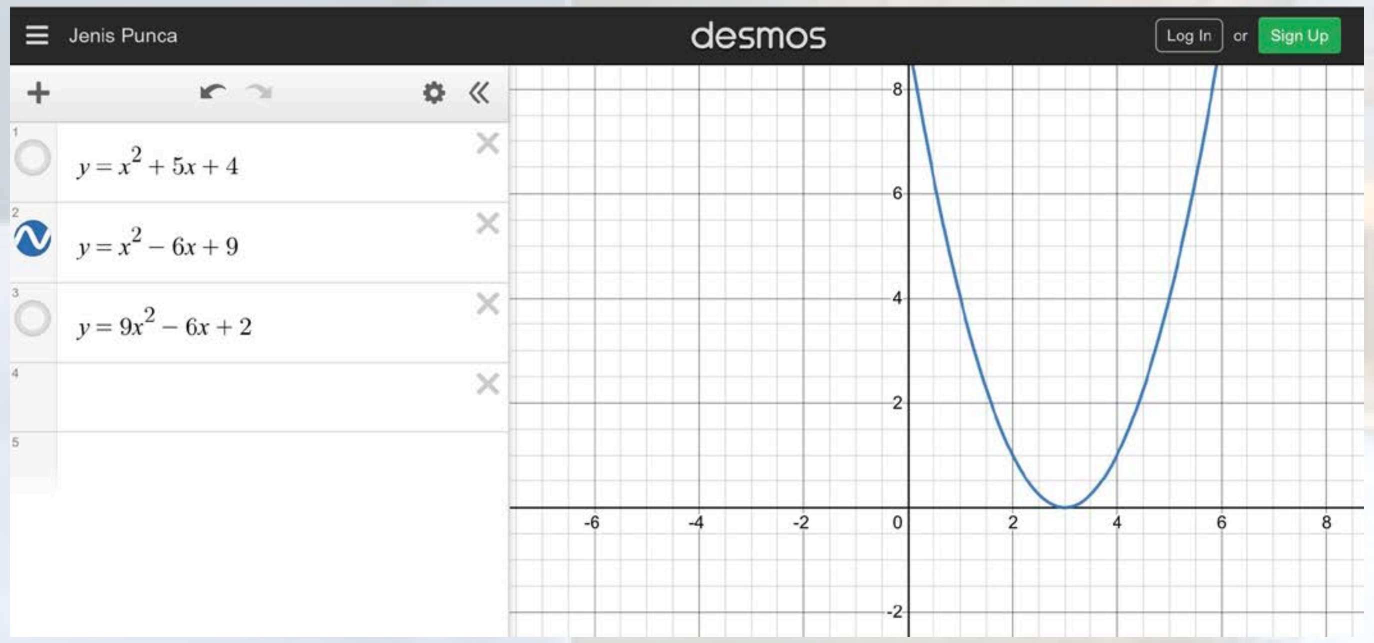
$a = 1, b = -6, c = 9$

$b^2 - 4ac = (-6)^2 - 4(1)(9)$

$= 0$

$b^2 - 4ac = 0$

the equation has two equal real roots



$y = 9x^2 - 6x + 2$

$y = 0, \quad x^2 - 6x + 9 = 0$

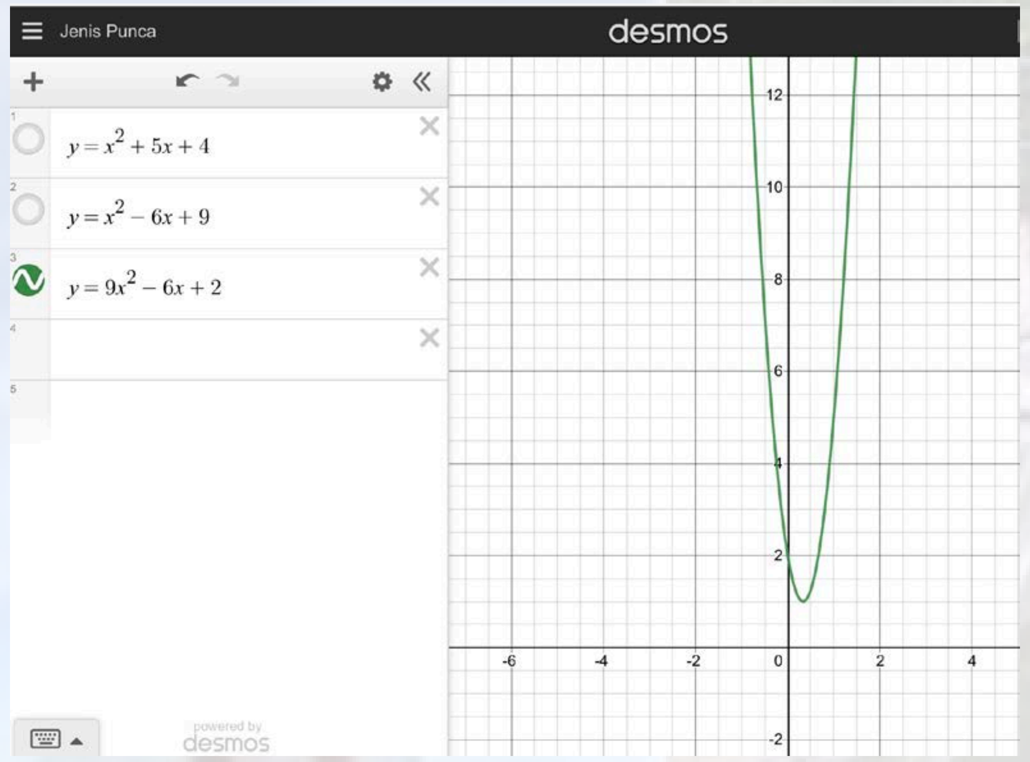
$a = 9, b = -6, c = 2$

$b^2 - 4ac = (-6)^2 - 4(9)(2)$

$= -36 < 0$

$b^2 - 4ac < 0$

the equation has no real roots



Type of Roots of Quadratic Equations

Example 7

Find the values of k or the range of values of k such that the equation $x^2 + kx = k - 8$ has

(a) two equal roots

(b) two real and different roots

(c) real roots

Solution 7(a):

two equal roots $b^2 - 4ac = 0$

(a) $x^2 + kx = k - 8$

$x^2 + kx - k + 8 = 0$

$a = 1,\ b = k,\ c = -k + 8$

$b^2 - 4ac = 0$

$(k)^2 - 4(1)(-k + 8) = 0$

$k^2 + 4k - 32 = 0$

$(k - 4)(k + 8) = 0$

$k = 4,\ k = -8$

Type of Roots of Quadratic Equations

Solution 7(b):

two real and different roots

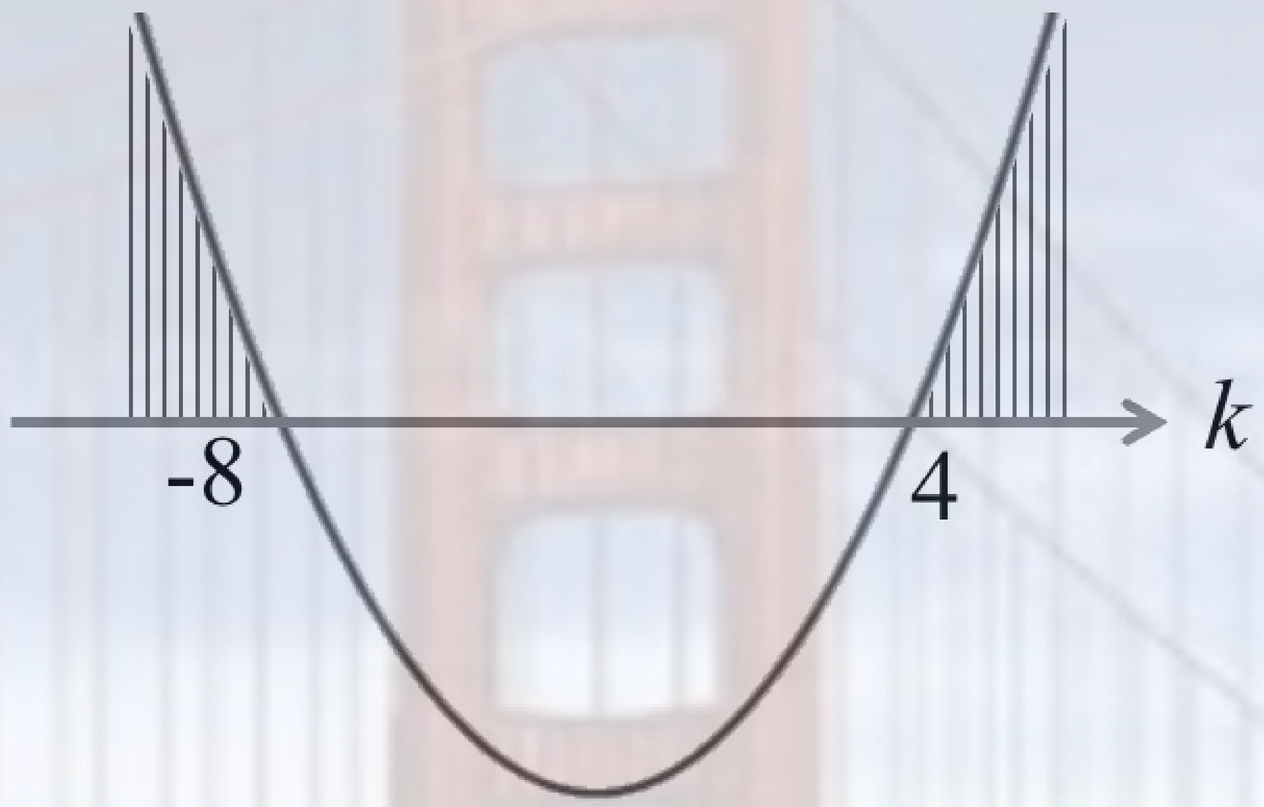
(b)

$$b^2 - 4ac > 0$$

$$(k)^2 - 4(1)(-k+8) > 0$$

$$k^2 + 4k - 32 > 0$$

$$(k-4)(k+8) > 0$$



$$k < -8,\ k > 4$$

real roots

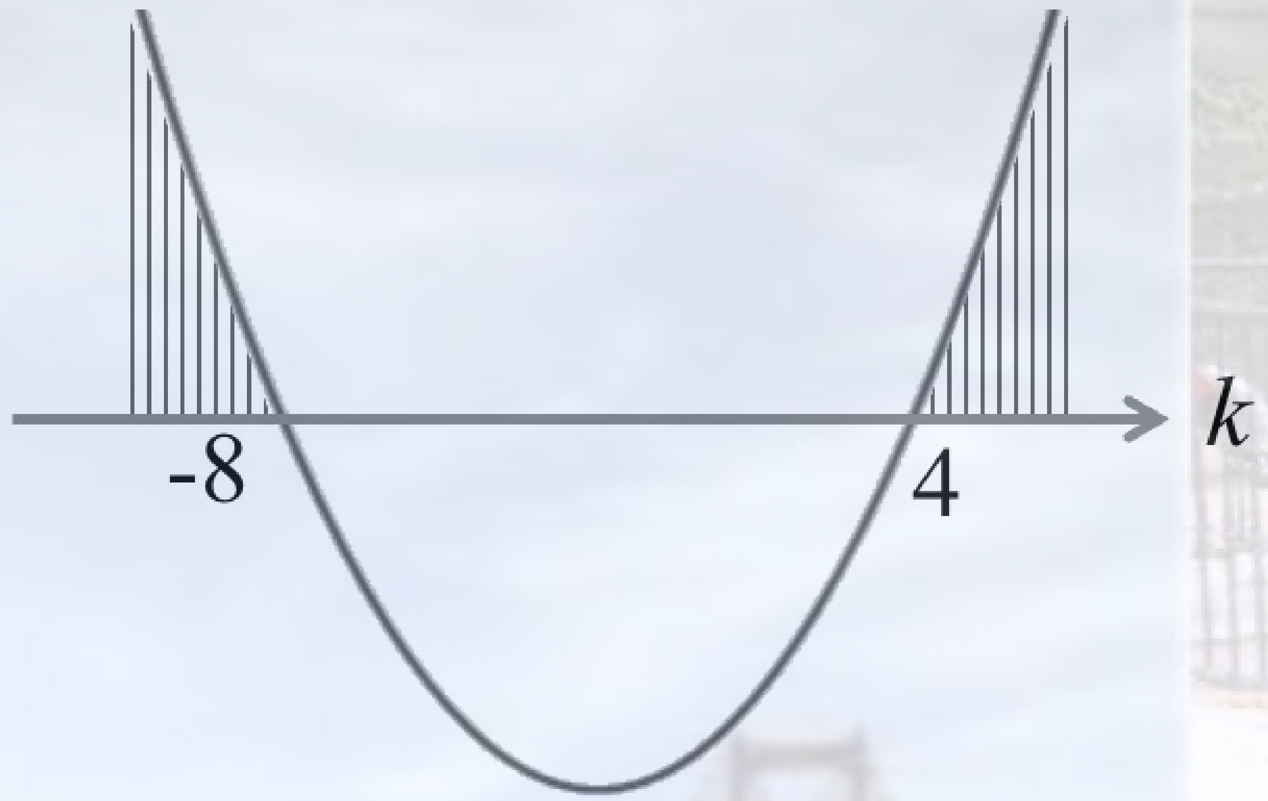
(c)

$$b^2 - 4ac \geq 0$$

$$(k)^2 - 4(1)(-k+8) \geq 0$$

$$k^2 + 4k - 32 \geq 0$$

$$(k-4)(k+8) \geq 0$$



$$k \leq -8,\ k \geq 4$$

Find the values of k or the range of values of k such that the equation $x^2 + kx = k - 8$ has

(a) two equal roots

(b) two real and different roots

(c) real roots

Type of Roots of Quadratic Equations

Example 8

Given $3hx^2 - 7kx + 3h = 0$ has two real and equal roots, where h and k are positive. Find the ratio $h:k$.

Solution 8 :

$h:k$ can be written as $\frac{h}{k}$

$$b^2 - 4ac = 0$$

$$a = 3h,\ b = -7k,\ c = 3h$$

$$(-7k)^2 - 4(3h)(3h) = 0$$

$$49k^2 - 36h^2 = 0$$

$$49k^2 = 36h^2$$

$$\frac{49}{36} = \frac{h^2}{k^2}$$

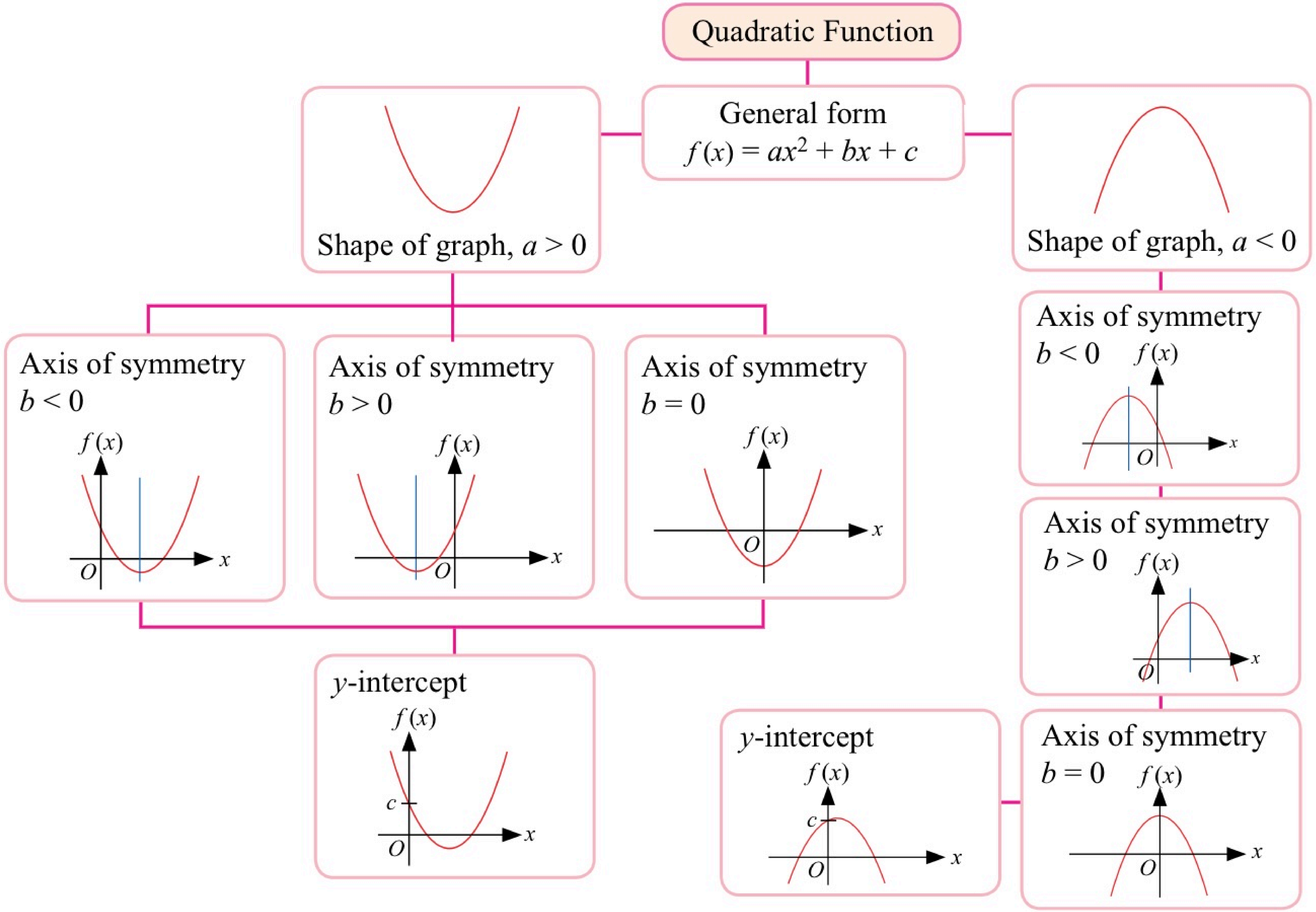
$$\frac{h^2}{k^2} = \frac{7^2}{6^2}$$

$$\frac{h}{k} = \frac{7}{6}$$

$$h:k = 7:6$$

Quadratic Functions

Prior Knowledge (Mathematics Form 4)

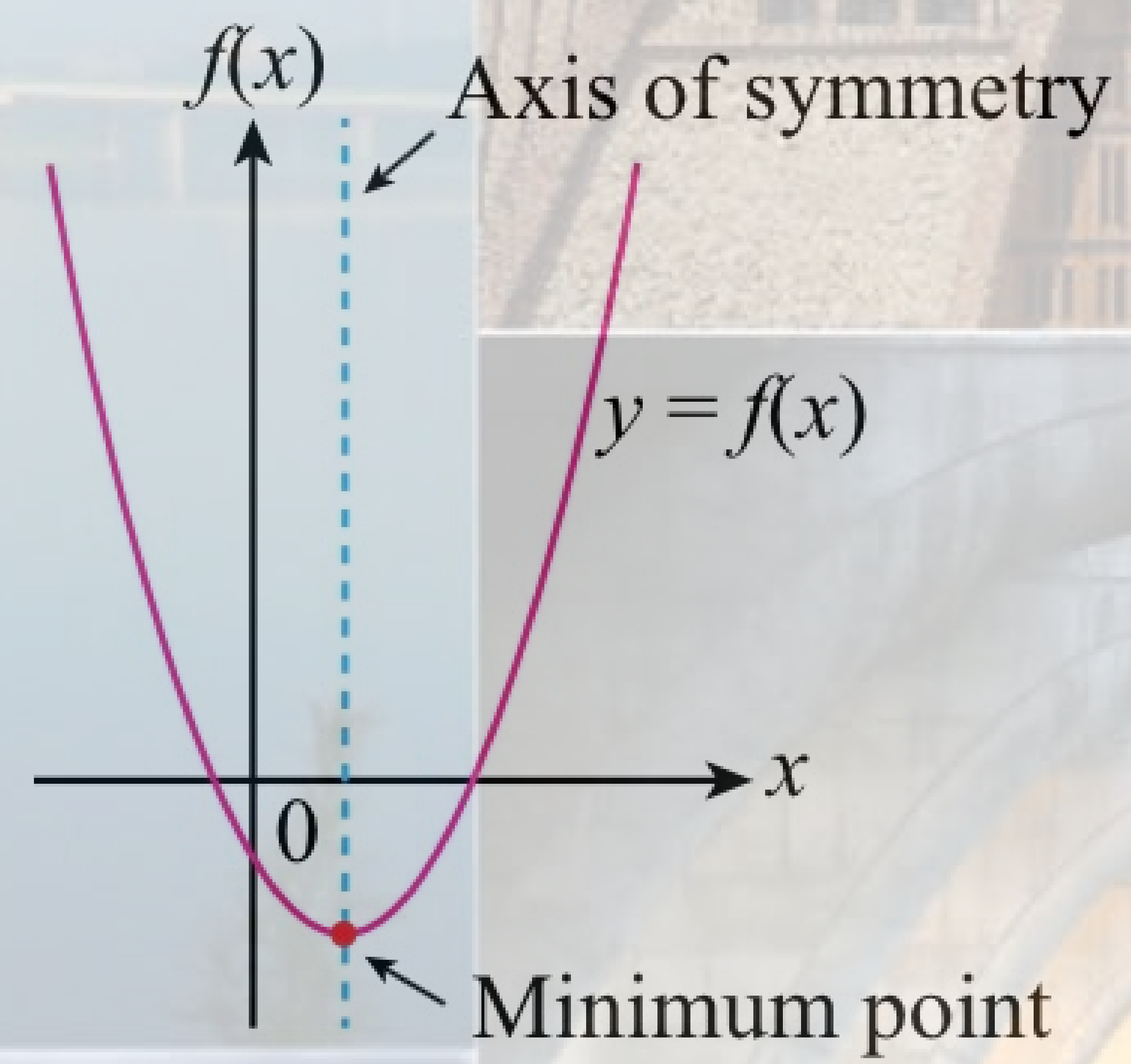


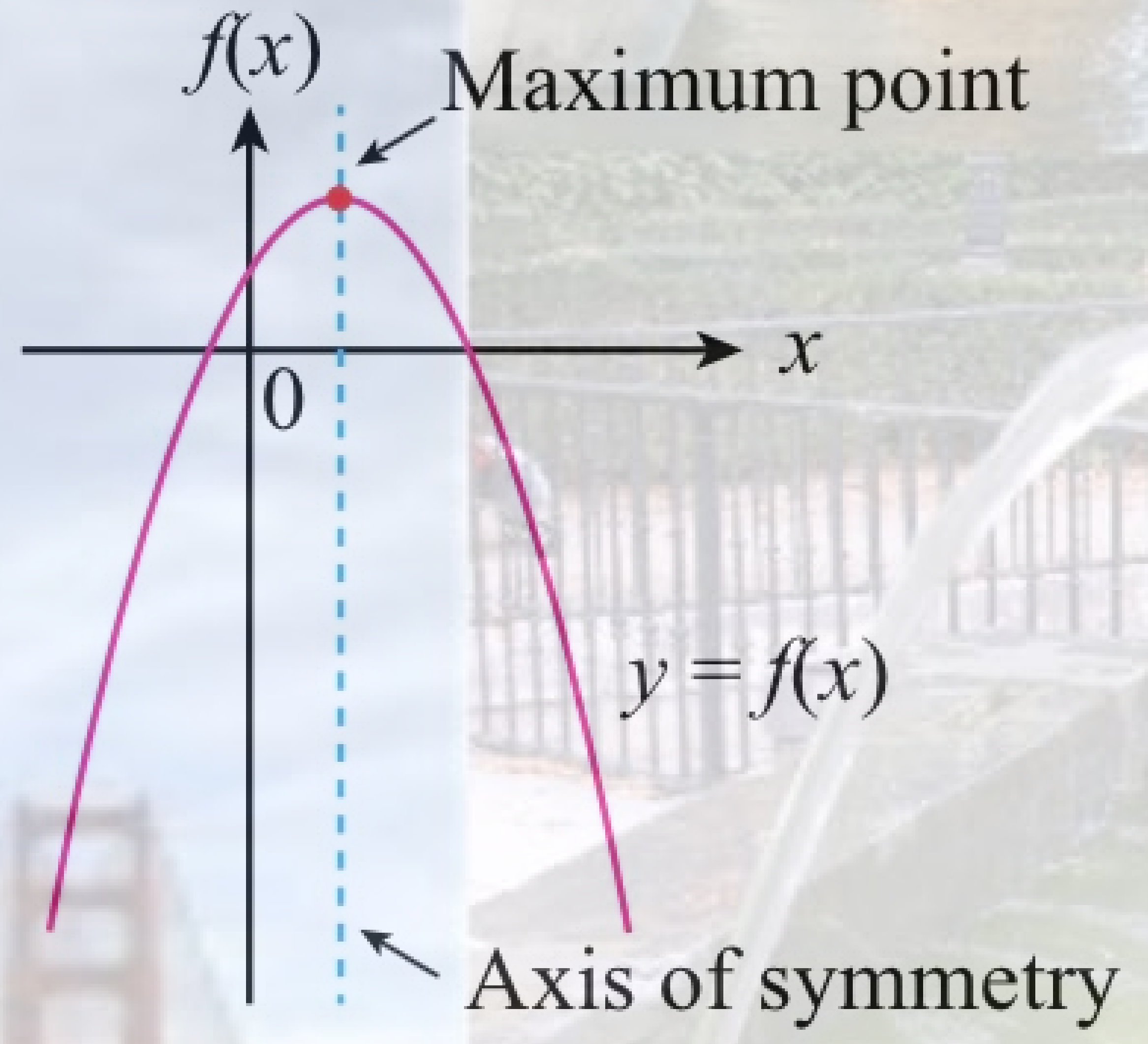
Quadratic Functions

General form of a quadratic function

$$f(x) = ax^2 + bx + c$$

where a, b and c are constant and $a \neq 0$





Analysing the effect of changes of a, b and c towards the shape and position of the graph for $f(x) = ax^2 + bx + c$

Example 9

http://bit.ly/324HT0w

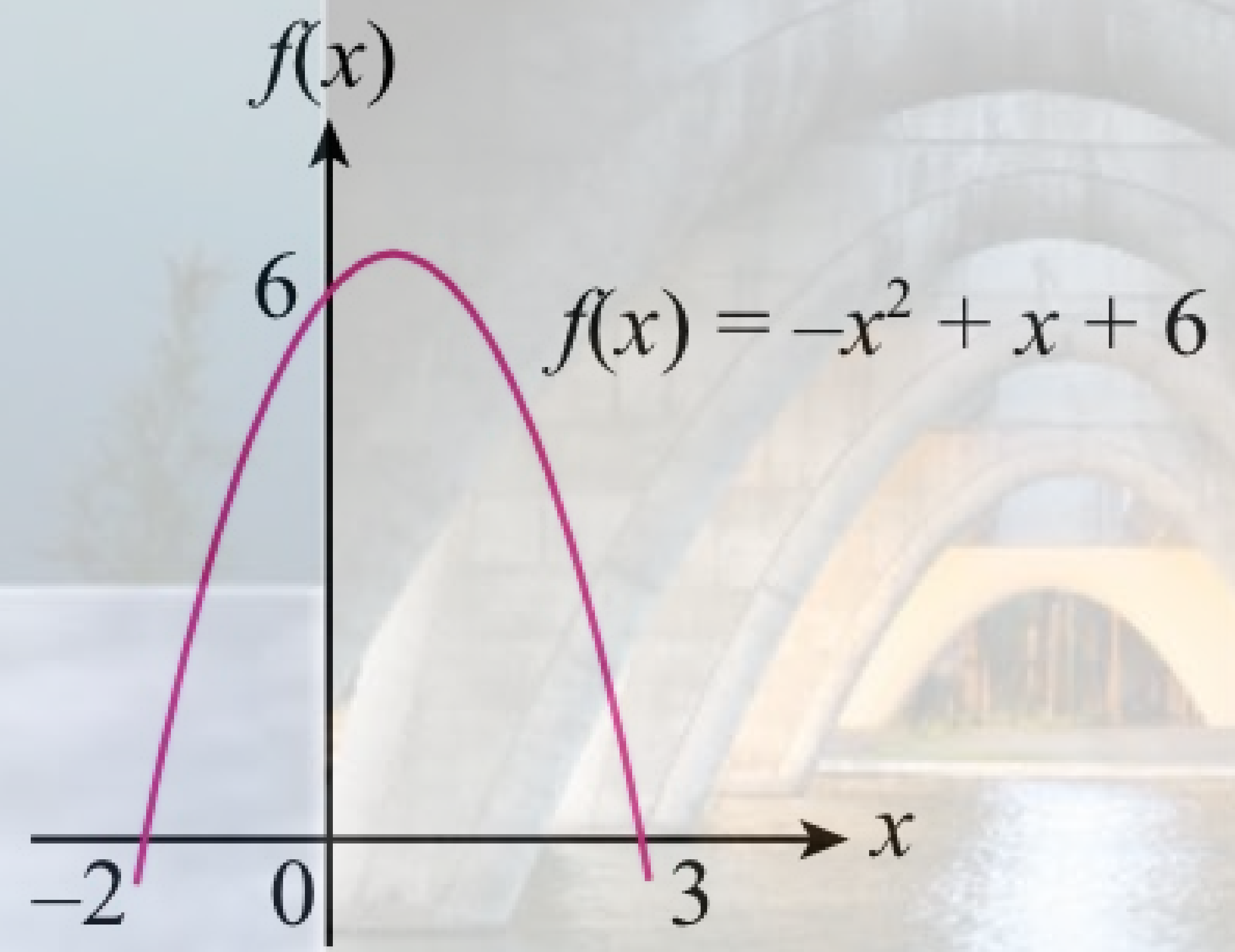
The diagram on the right shows the graph for $f(x) = -x^2 + x + 6$, where $a = -1$, $b = 1$ and $c = 6$. Sketch the graph of $f(x)$ formed when the following values change.

(a) The value of a changes to

(i) -3 (ii) $-\frac{1}{4}$,

(b) The value of b changes to -1,

(c) The value of c changes to -2.



Analysing the effect of changes of a, b and c towards the shape and position of the graph for *f*(*x*) = *ax*² + *bx* + *c*

Solution 9 :

The diagram on the right shows the graph for $f(x) = -x^2 + x + 6$, where $a = -1$, $b = 1$ and $c = 6$. Sketch the graph of $f(x)$ formed when the following values change.

(a) The value of a changes to

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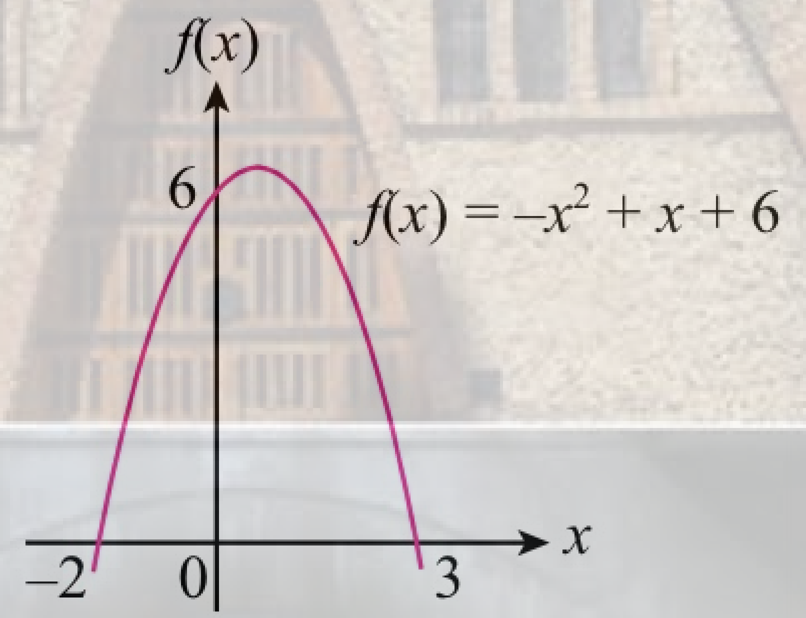
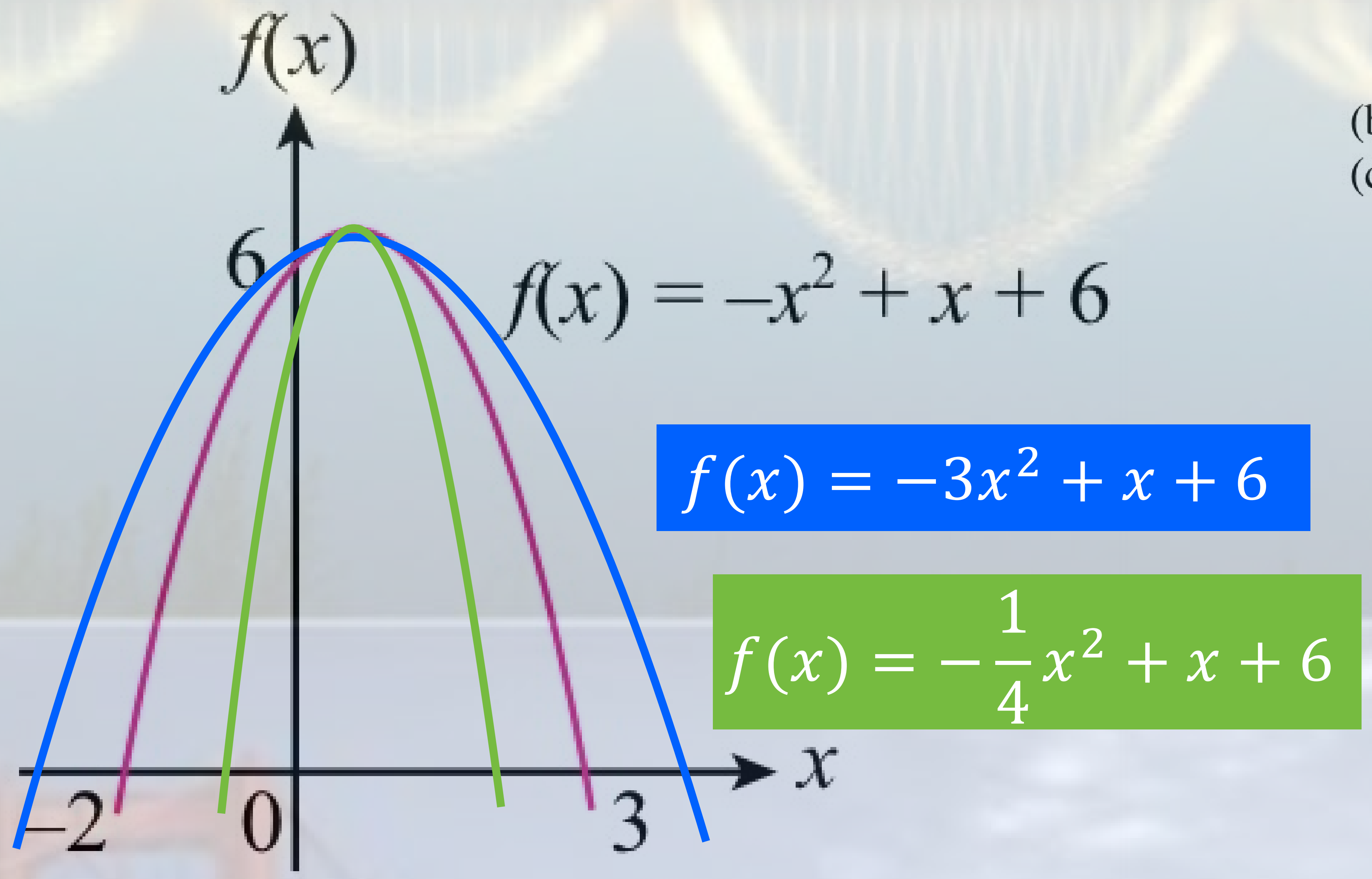
(b) The value of b changes to -1,

(c) The value of c changes to -2.

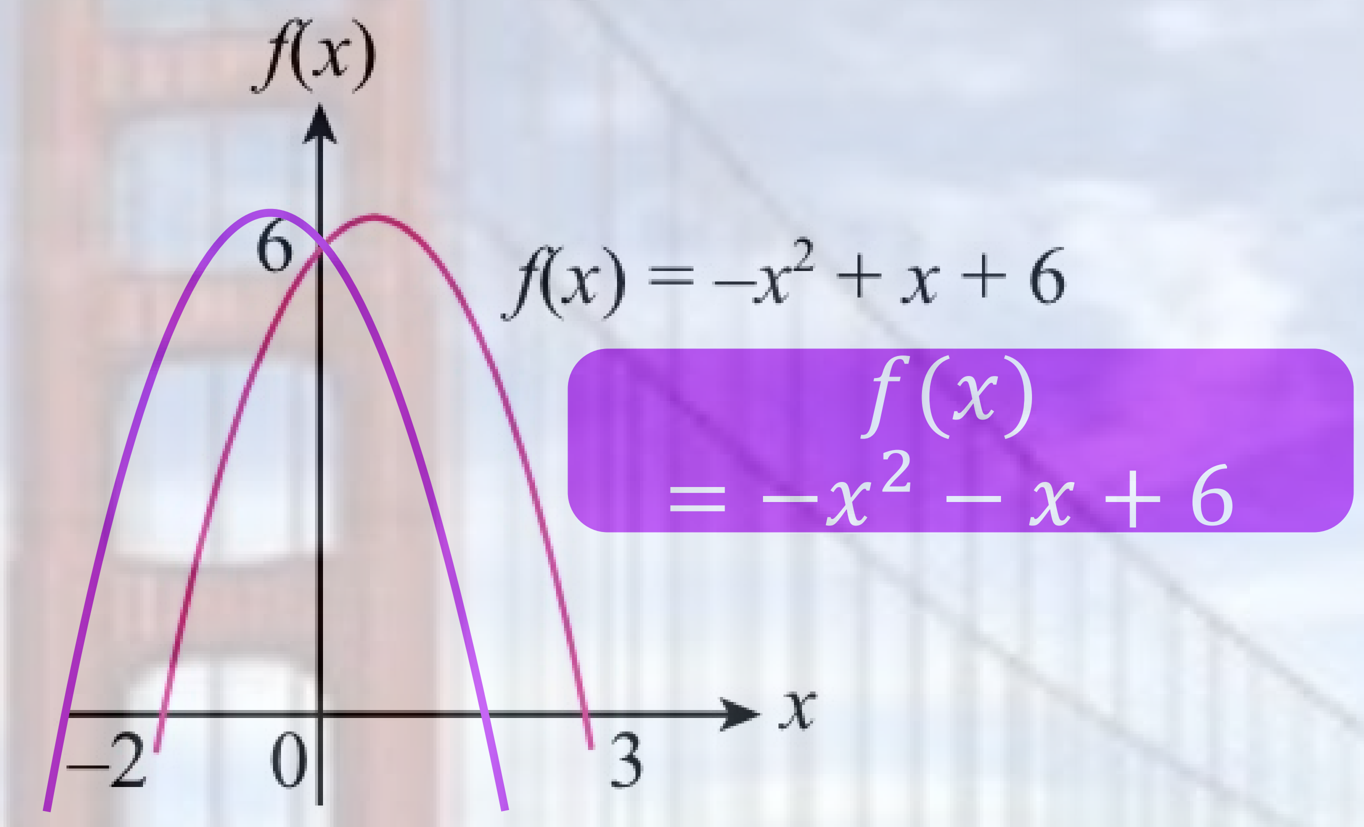


(a) (i)

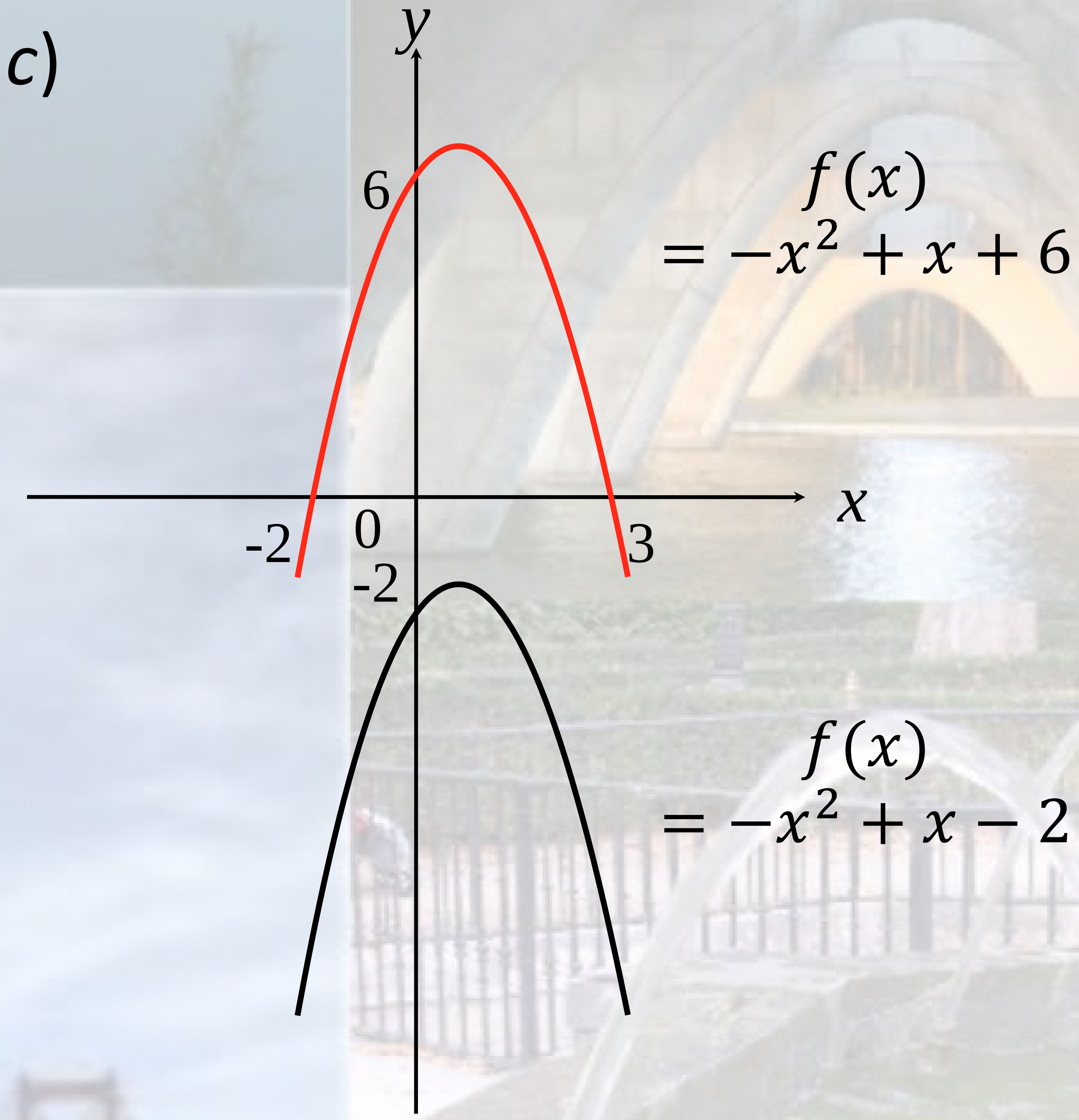
(ii)



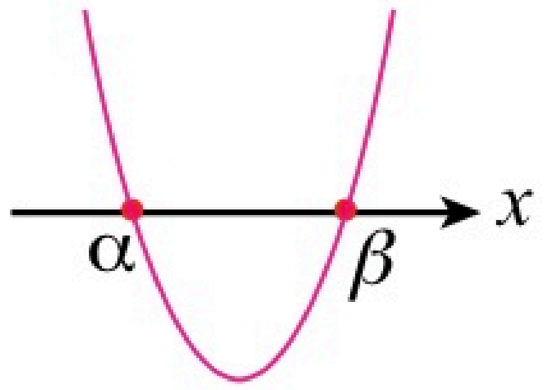
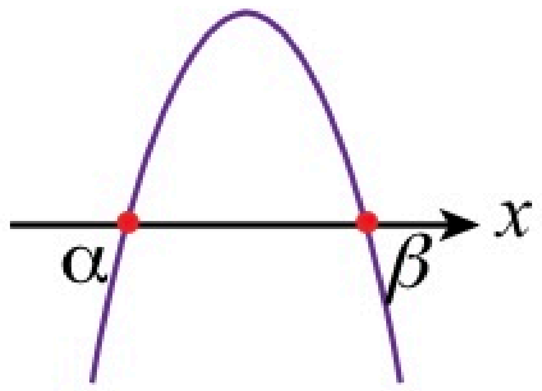
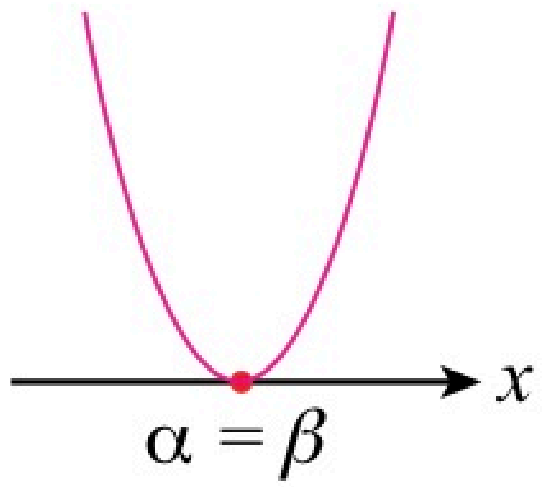
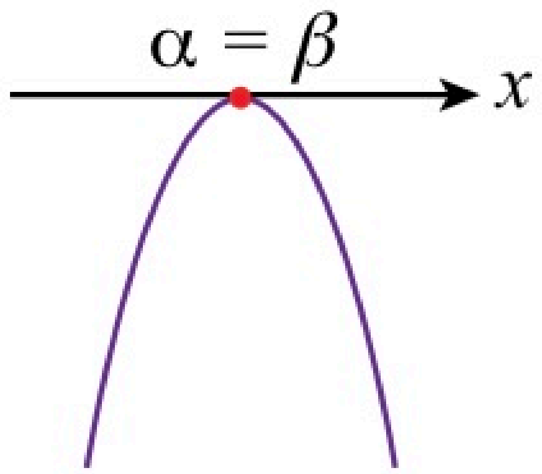
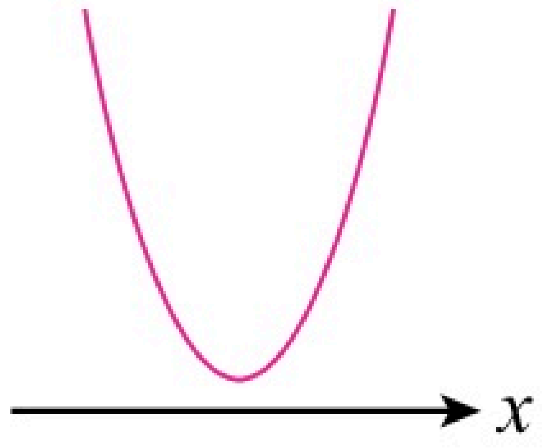
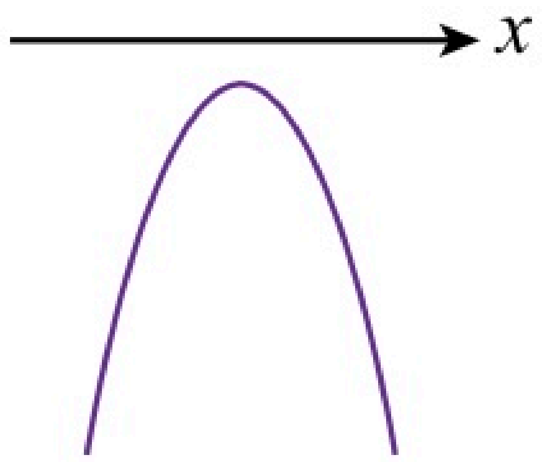
(b)



(c)



Relating the position of the graph of a quadratic function and the types of roots

Discriminant, $b^2 - 4ac$	Types of roots and position of graph	Position of graph of function $f(x) = ax^2 + bx + c$	
		$a > 0$	$a < 0$
$b^2 - 4ac > 0$	• Two real and different roots • The graph intersects the x-axis at two different points.		
$b^2 - 4ac = 0$	• Two real and equal roots • The graph touches the x-axis at one point only.		
$b^2 - 4ac < 0$	• No real roots • The graph does not intersect at any point on the x-axis.		

Example 10

(*a*) Find the values of m such that the x-axis is the tangent to the graph of a quadratic function $f(x) = (m+1)x^2 + 4(m-2)x + 2m$

(*b*) Find the range of values of k if the graph of a quadratic function $f(x) = x^2 + 5x + 3 - k$ has no intercept.

Solution 10 :

(*a*) $b^2 - 4ac = 0$

$a = m + 1,\ b = 4m - 8,\ c = 2m$

$(4m-8)^2 - 4(m+1)(2m) = 0$

$16m^2 - 64m + 64 - 8m^2 - 8m = 0$

$8m^2 - 72m + 64 = 0$

$m^2 - 9m + 8 = 0$

$(m-1)(m-8) = 0$

$m = 1,\quad m = 8$

(b) $b^2 - 4ac < 0$

$(5)^2 - 4(1)(3-k) < 0$

$25 - 12 + 4k < 0$

$13 + 4k < 0$

$4k < -13$

$k < -\frac{13}{4}$

Making relation between the vertex form of a quadratic function $f(x) = a(x - h)^2 + k$ with the other forms of quadratic functions

General form

$$f(x)=ax^2+bx+c$$

$$=a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$$

$$=a\left[x^2+\frac{b}{a}x+\left(\frac{\frac{b}{a}}{2}\right)^2-\left(\frac{\frac{b}{a}}{2}\right)^2+\frac{c}{a}\right]$$

$$=a\left[x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}\right]$$

$$=a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c}{a}\right]$$

$$f(x)=a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c$$

Vertex form

$$f(x)=a(x-h)^2+k$$

$$f(x)=a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c$$

$$-h=\frac{b}{2a} \qquad k=-\frac{b^2}{4a}+c$$

$$h=-\frac{b}{2a} \qquad =c-\frac{b^2}{4a}$$

Intercept form

$$f(x)=a(x-p)(x-q)$$

Example 11

Express the quadratic function $f(x) = x^2 - 4x - 5$ in the form of

(a) vertex form $f(x) = a(x - h)^2 + k$,

(b) intercept form $f(x) = a(x - p)(x - q)$.

Solution 11 :

General form

$$f(x) = x^2 - 4x - 5$$

$$= x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 - 5$$

completing the square

$$= x^2 - 4x + (-2)^2 - (-2)^2 - 5$$

$$= (x-2)^2 - (4) - 5$$

$$= (x-2)^2 - 9$$

$$f(x) = (x-2)^2 - 9$$

Vertex form

Making relation between the vertex form of a quadratic function *f*(*x*) = *a*(*x* - *h*)² + *k* with the other forms of quadratic functions

Solution 11 :

Vertex form

Alternative method

$$f(x)=x^2-4x-5$$

$$a=1,\ b=-4,\ c=-5$$

$$f(x)=a(x-h)^2+k$$

$$h=-\frac{b}{2a} \qquad k=c-\frac{b^2}{4a}$$

$$h=-\frac{(-4)}{2(1)} \qquad h=2$$

$$k=-5-\frac{(-4)^2}{4(1)}=-5-\frac{16}{4}$$

$$k=-5-4$$

$$k=-9$$

$$f(x)=(x-2)^2-9$$

Intercept form

$$f(x)=(x-2)^2-9$$

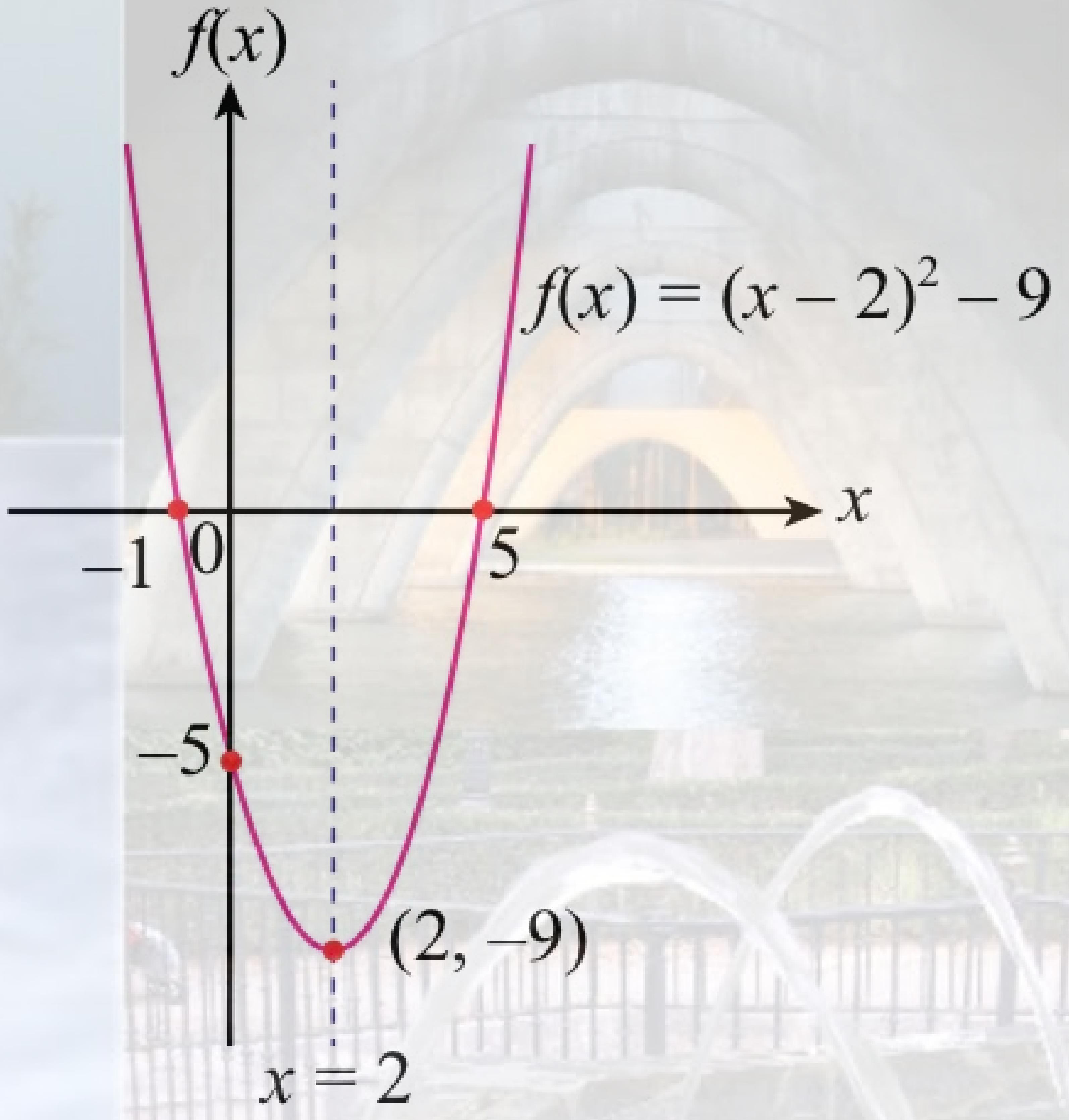
$$=(x-2)^2-(3)^2$$

$$=(x-2+3)(x-2-3)$$

$$f(x)=(x+1)(x-5)$$

$$a^2-b^2=(a+b)(a-b)$$

$$a=x-2 \quad b=3$$



Making relation between the vertex form of a quadratic function $f(x) = a(x - h)^2 + k$ with the other forms of quadratic functions

Example 12

The diagram on the right shows the graph of $f(x) = -3(x - 2)^2 + 4$ where $a = -3$, $h = 2$ and $k = 4$.

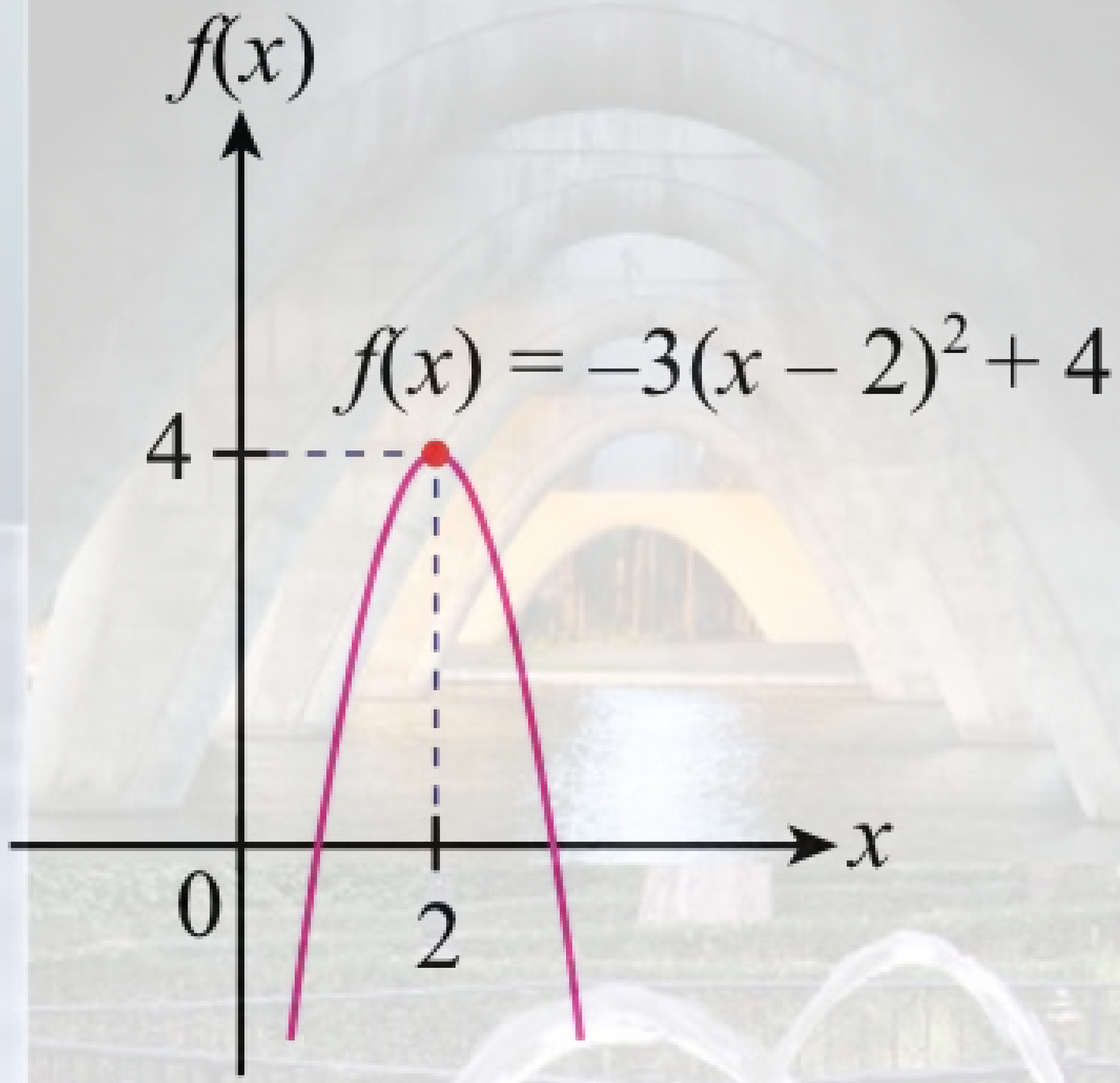
(a) Determine the coordinate of maximum point and the equation of the axis of symmetry.

(b) Make generalisation on the shape and position of the graph when the following values change. Hence, sketch the graphs.

(i) The value of a changes to -10.

(ii) The value of h changes to 5.

(iii) The value of k changes to -2.



Analysing the effect of change of a, h and k on the shape and position of graph $f(x) = a(x - h)^2 + k$

The diagram on the right shows the graph of $f(x) = -3(x - 2)^2 + 4$ where $a = -3$, $h = 2$ and $k = 4$.

(a) Determine the coordinate of maximum point and the equation of the axis of symmetry.

(b) Make generalisation on the shape and position of the graph when the following values change. Hence, sketch the graphs.

(i) The value of a changes to -10.

(ii) The value of h changes to 5.

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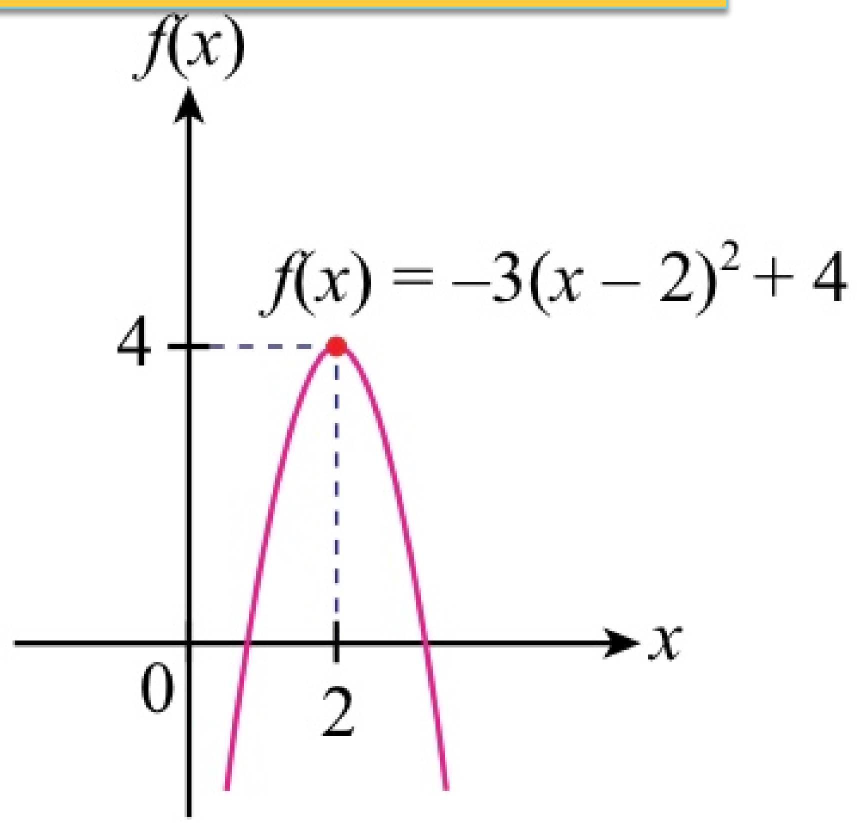
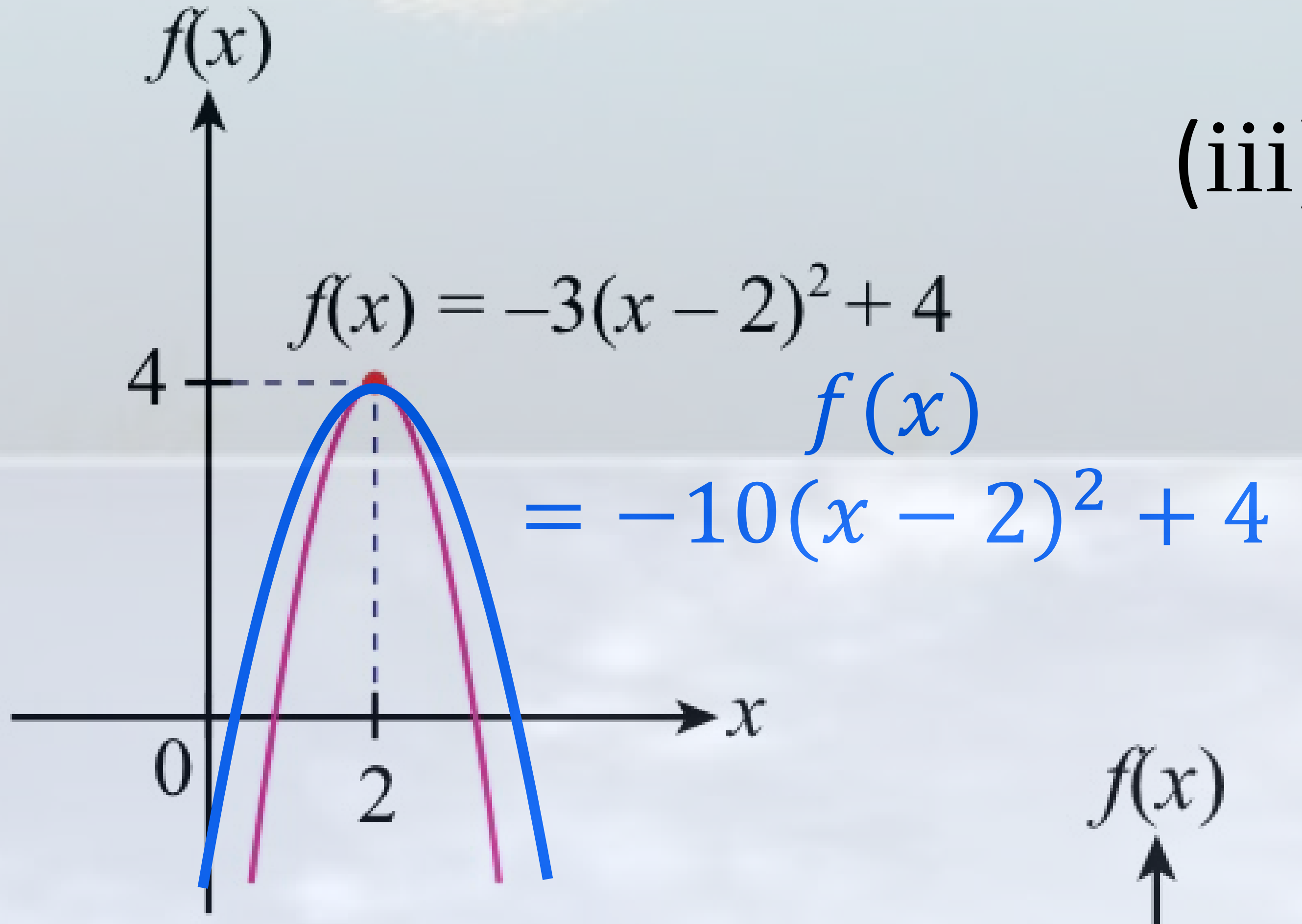


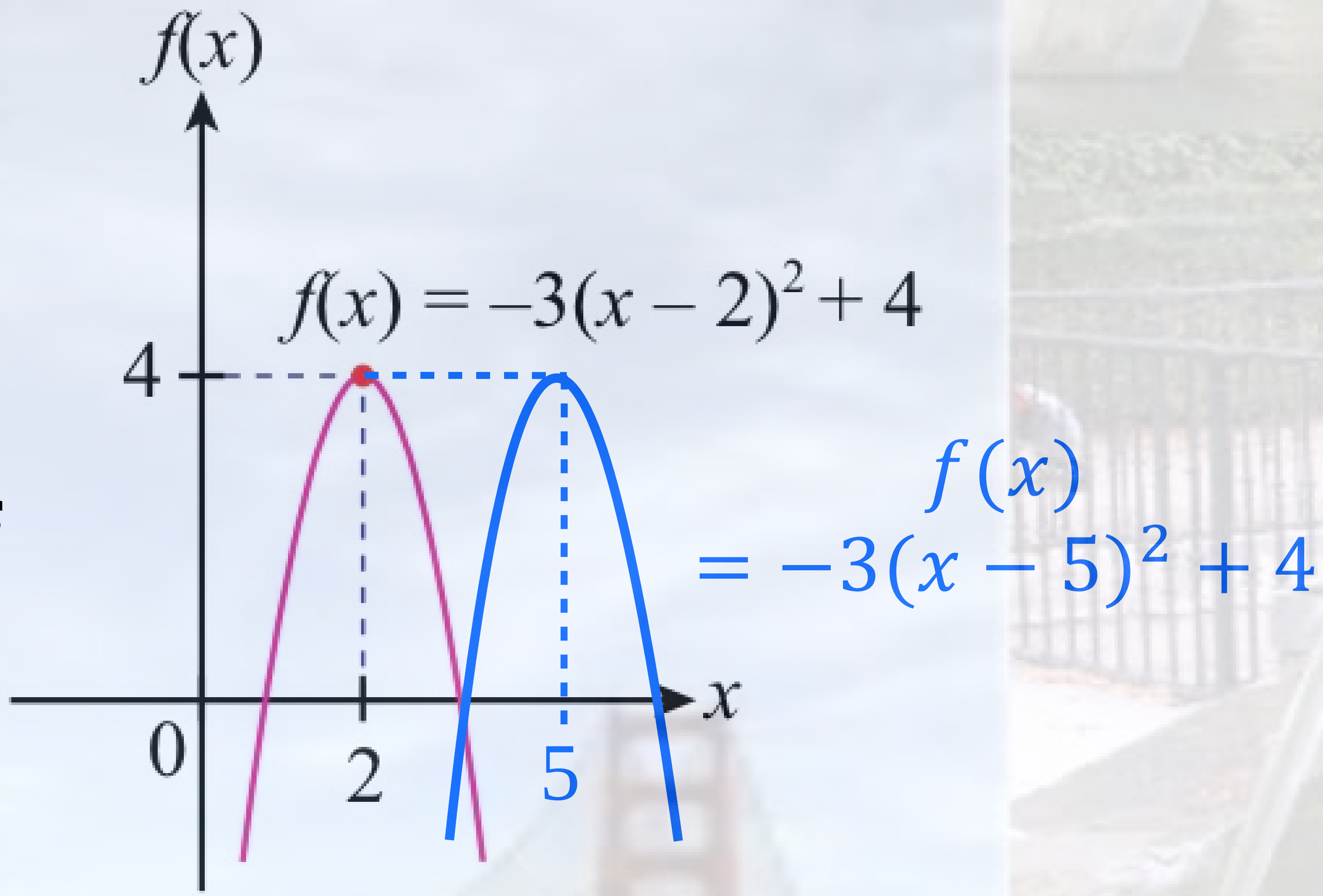
Solution 12 :

(*a*) The maximum point is (2, 4) and the equation for the axis of symmetry is $x = 2$.

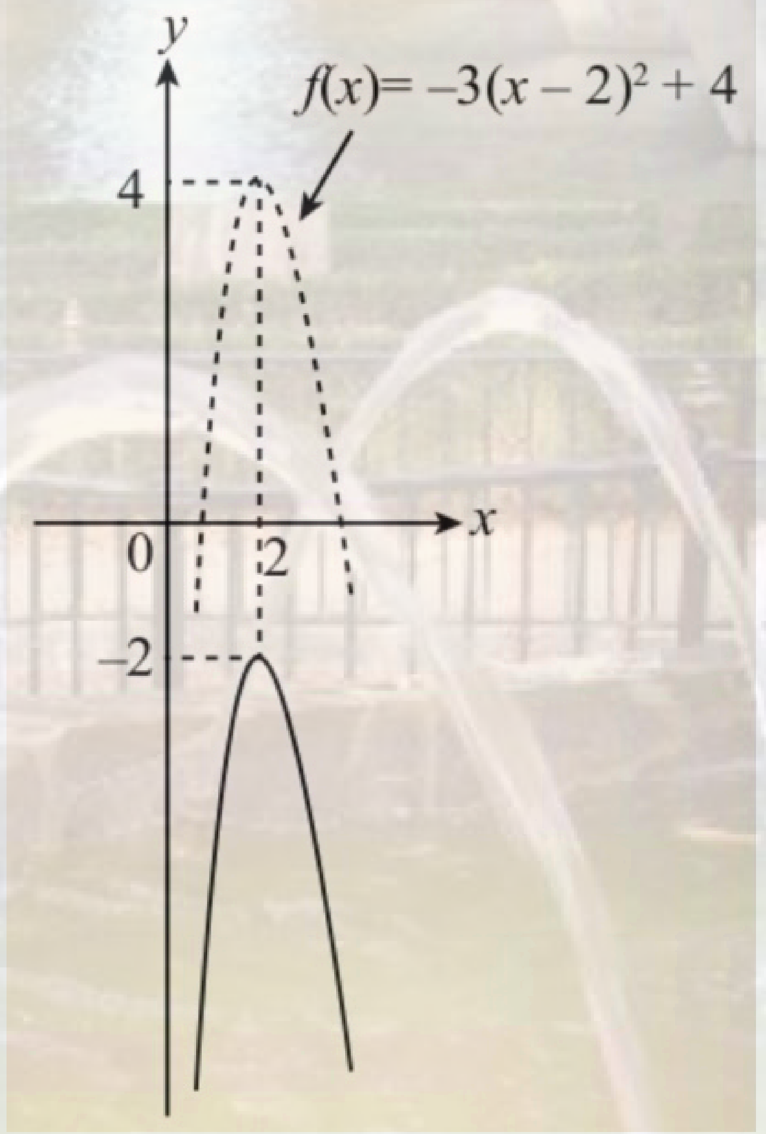
(b) (i) When the value of a changes from -3 to -10, the width of the graph decreases. The axis of symmetry, $x = 2$ and the maximum value, 4 does not change.



(ii) When the value of h changes from 2 to 5, the graph with the same shape moves horizontally 3 units to the right. The equation of the axis of symmetry $x = 5$ and its maximum value does not change which is 4.



(iii) When the value of k changes from 4 to -2, the width of the same shape moves vertically 6 units downwards. Its maximum value becomes -2 and the axis of symmetry does not change.



Sketching the graph of quadratic function

1. Identify the value of a to determine the shape of the graph of a quadratic function.
2. Find the value of discriminant, $b^2 - 4ac$ to determine the position of the graph.
3. Determine the vertex.
4. Determine the intersection point on the x-axis by solving the equation of quadratic function $f(x) = 0$.
5. Find the value of $f(0)$ to determine the y-intercept.
6. Plot the points obtained on the Cartesian plane and draw a smooth parabola that is symmetrical at the vertical line passing through the vertex.

Sketching the graph of quadratic function

Example 13

Sketch the graph of quadratic function $f(x) = 2x^2 + 3x - 2$.

Solution 13:

$a = 2 > 0$, minimum graph

$b^2 - 4ac = (3)^2 - 4(2)(-2)$

$= 25$

The curve intersect at *x*-axis at two different points

Minimum point is $\left(-\frac{3}{4}, -\frac{25}{8}\right)$

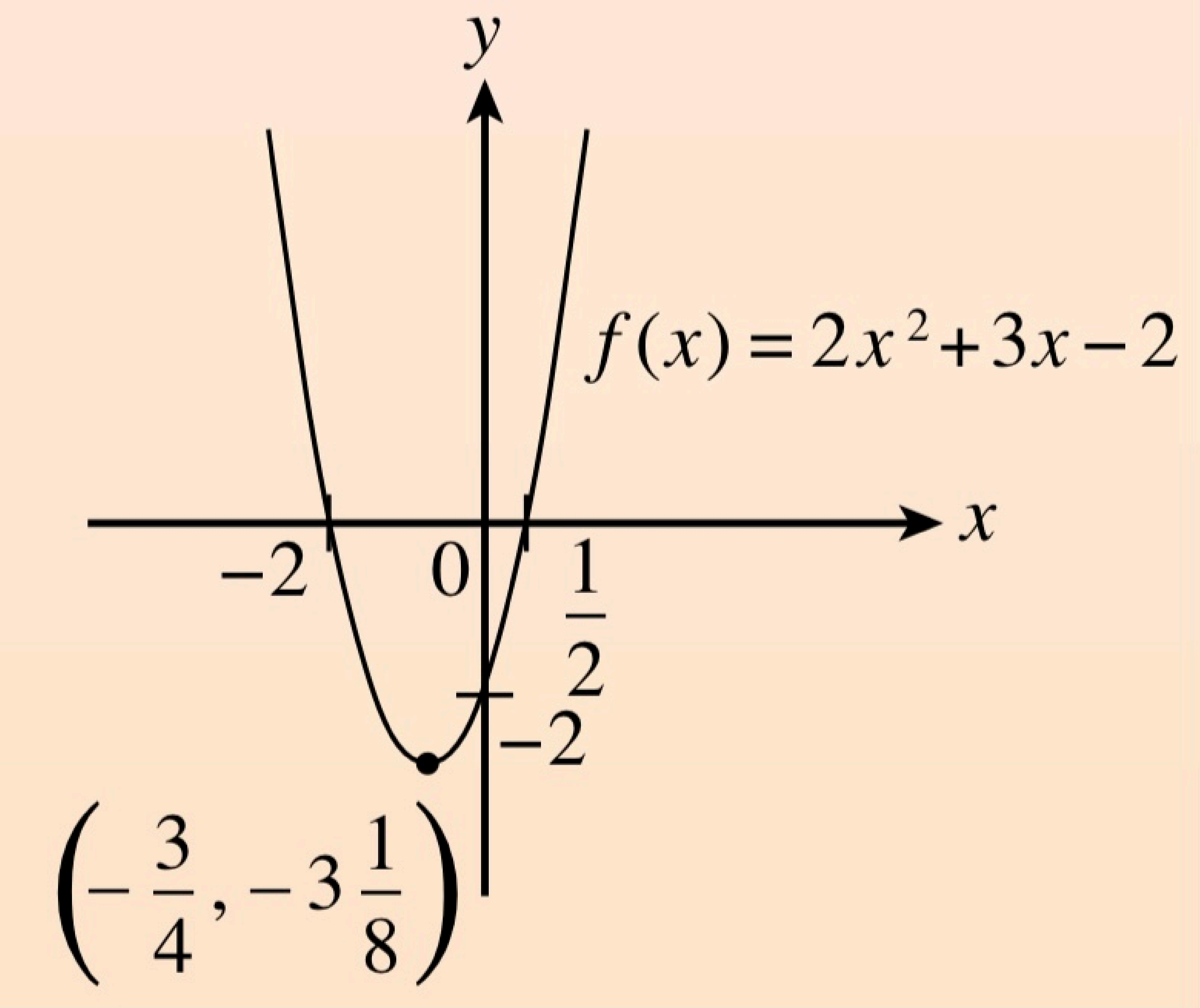
Axis of symmetry , $x = -\frac{3}{4}$

$$2x^2 + 3x - 2 = 0$$

$$(2x-1)(x+2) = 0$$

$$x = \frac{1}{2}, \quad x = -2$$

$$f(0) = 2(0)^2 + 3(0) - 2 = -2$$



$$f(x) = 2x^2 + 3x - 2$$

$$= 2\left(x^2 + \frac{3}{2}x - 1\right)$$

$$= 2\left[x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - 1\right]$$

$$= 2\left[\left(x + \frac{3}{4}\right)^2 - \frac{9}{16} - 1\right]$$

$$= 2\left(x + \frac{3}{4}\right)^2 - \frac{9}{8} - 2$$

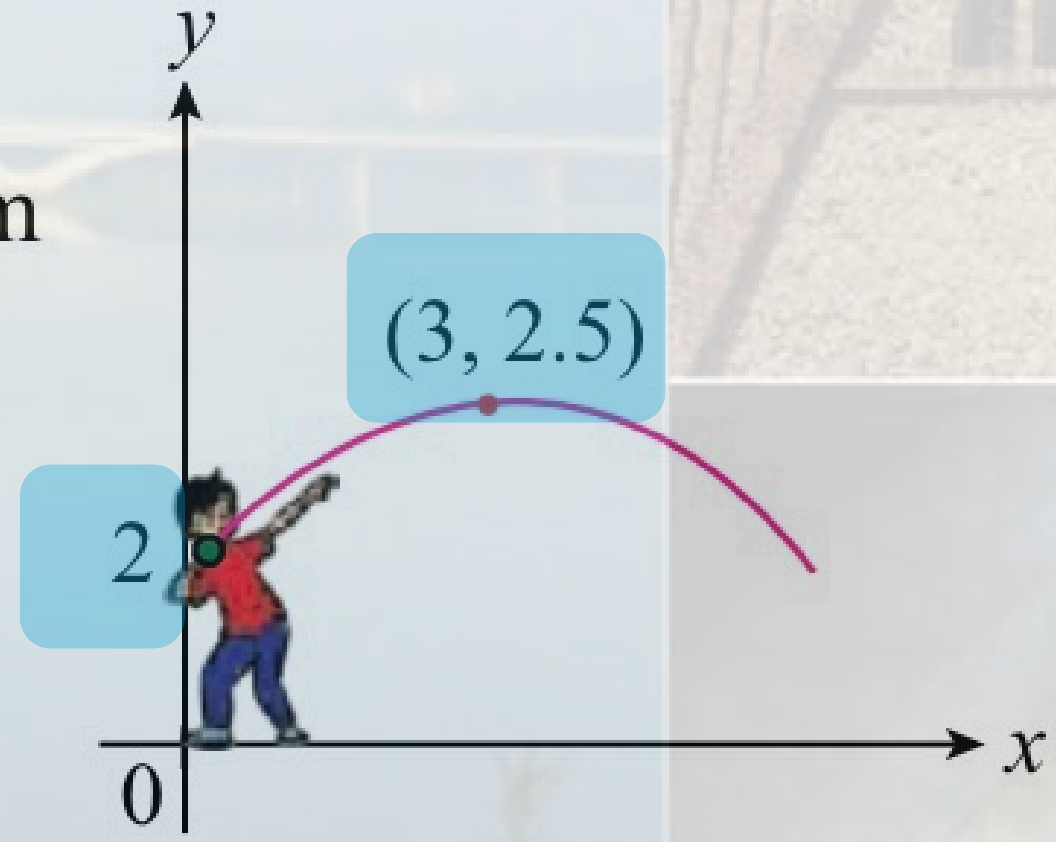
$$= 2\left(x + \frac{3}{4}\right)^2 - \frac{25}{8}$$

Example 14

The path of a shot put thrown by Krishna in a competition can be represented by the quadratic function graph as shown in the diagram on the right. The shot put is thrown at a height of 2 m and the path passes through the maximum point (3, 2.5).

(a) Express the equation of the path of shot put in the form $y = a(x - h)^2 + k$, where a, h and k are constants.

(b) Find the maximum distance of the horizontal throw, in metres, by Krishna.



Solution 14:

(a)

$$y=a(x-h)^2+k$$

$$y=a(x-3)^2+2.5$$

$$h=3 \quad k=2.5$$

$$2=a(0-3)^2+2.5$$

$$9a=-\frac{1}{2}$$

$$a=-\frac{1}{18}$$

(b)

$$y=-\frac{1}{18}(x-3)^2+2.5$$

$$0=-\frac{1}{18}(x-3)^2+2.5$$

$$-\frac{1}{8}(x-3)^2+2.5=0$$

$$\frac{1}{18}(x-3)^2=2.5$$

$$(x-3)^2=45$$

$$x-3=\pm\sqrt{45}$$

$$x=3+\sqrt{45} \qquad x=3-\sqrt{45}$$

$$x=9.708 \qquad x=-3.708$$

$$x=9.708$$

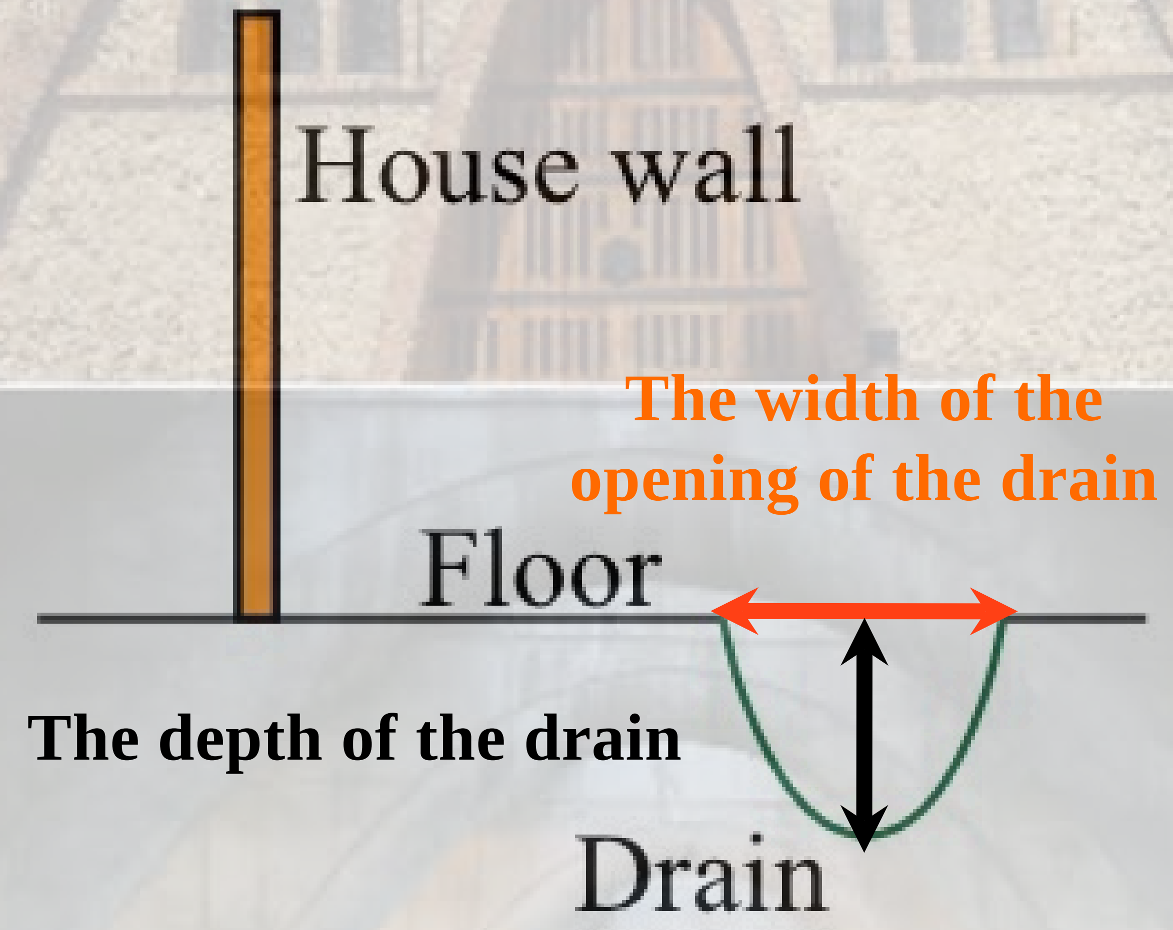
Mastery Practice

Example 15

The diagram on the right shows the cross section of a drain around the house. If the shape of the drain is represented by the equation $y = \frac{1}{5}x^2 - 24x + 700$, find

(a) the width of the opening of the drain,

(b) the minimum depth of the drain.



Solution 15:

(a) $y = \frac{1}{5}x^2 - 24x + 700$

$\frac{1}{5}x^2 - 24x + 700 = 0$

$x^2 - 120x + 3500 = 0$

$(x-50)(x-70) = 0$

$x = 50 \quad x = 70$

The width of the opening of the drain $= 70 - 50$

$= 20$

(b) The x coordinate of the minimum point $= \frac{50+70}{20} = 60$

When $x = 60$,

$y = \frac{1}{2}(60)^2 - 24(60) + 700$

$= -20$

The minimum depth of the drain is 20 units

SPM Practice

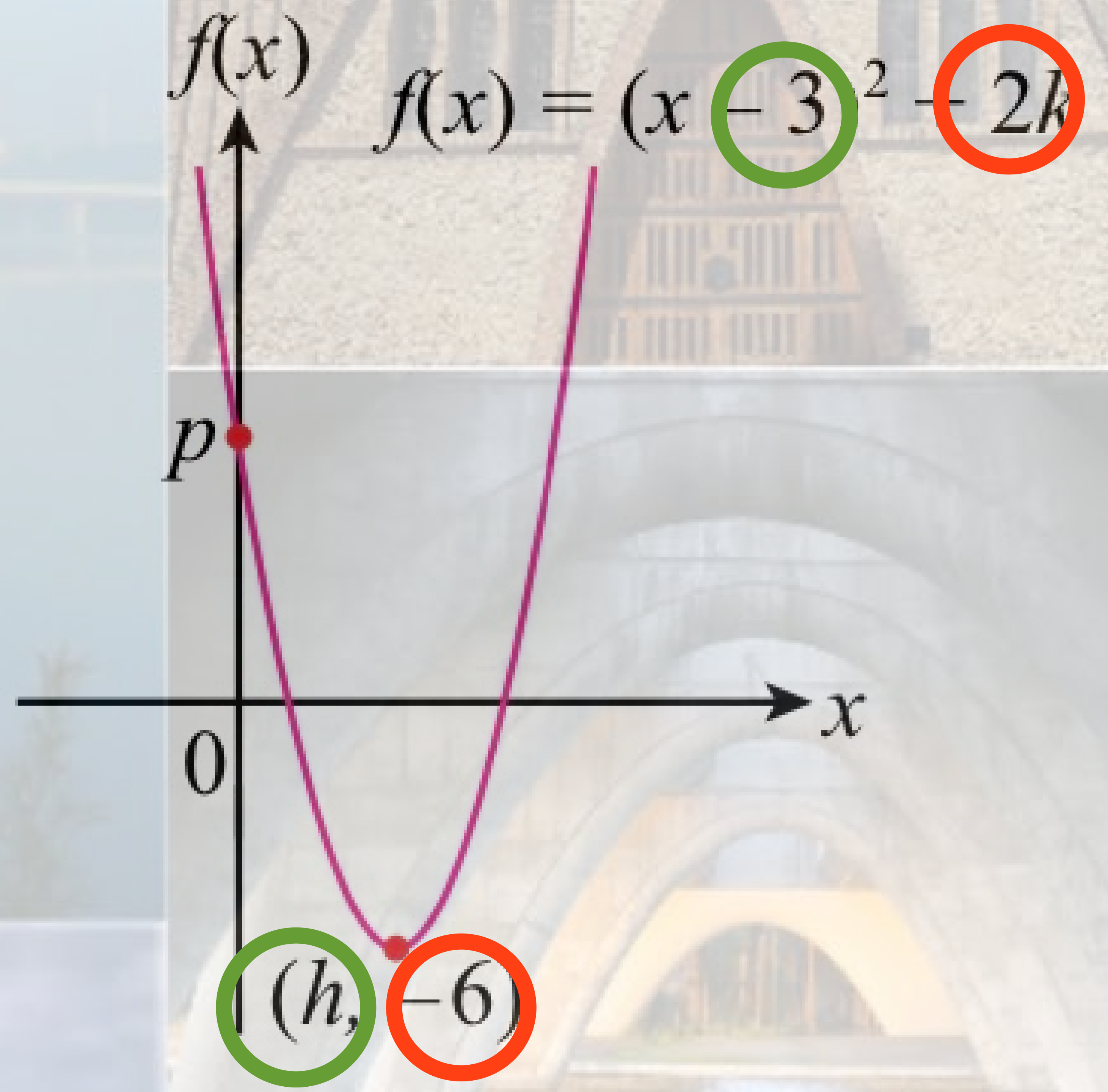
Example 16

The diagram on the right shows the graph of function $f(x) = (x - 3)^2 + 2k$, where k is a constant. Given $(h, -6)$ is the minimum point of the graph.

(a) State the values of h, k and p.

(b) If the graph moves 2 units to the right, determine the equation of the axis of symmetry for the curve.

(c) If the graph moves 5 units upwards, determine the minimum value.



Solution 16 :

(a) $h = 3$

$2k = -6$

$k = -3$

$f(x) = (x - 3)^2 + 2k$

$f(x) = (x - 3)^2 + 2(-3)$

$p = (0 - 3)^2 + 2(-3)$

$p = 9 - 6$

$p = 3$

(b) $(3, -6)$

$(3+2, -6)$

$(5, -6)$

$x = 5$

(c) $(3, -6)$

$(3, -6 + 5)$

$(3, -1)$

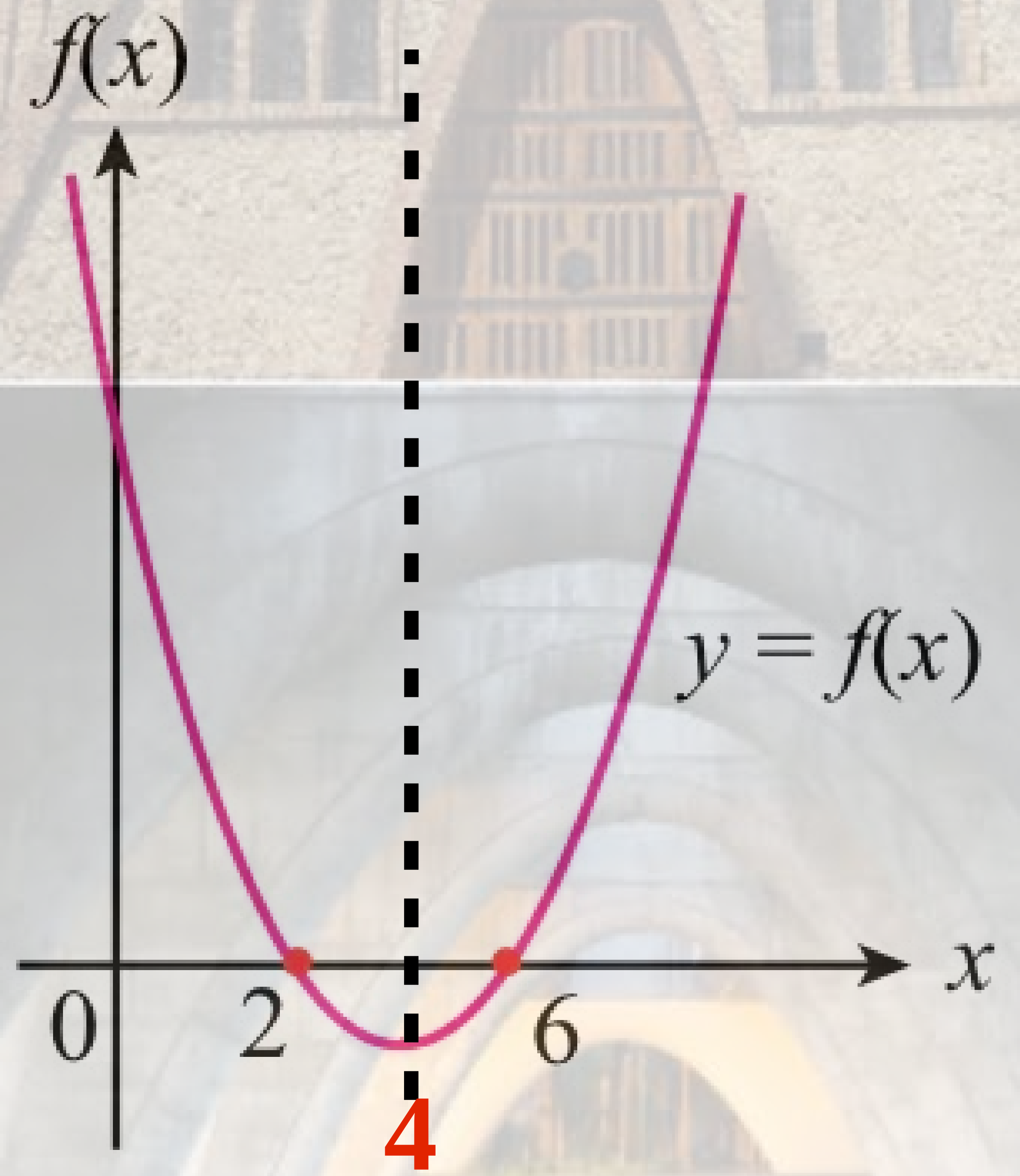
Minimum value = -1

SPM Practice

Example 17

The diagram on the right shows the graph of $f(x) = x^2 + bx + c$, where b and c are constants. Find,

(a) the values of b and of c,
(b) the coordinates of the minimum point,
(c) the range of values of x when $f(x)$ is negative,
(d) the maximum value when the graph is reflected in the x-axis.



Solution 17 :

(a) Sum of roots, SOR

$$-b = 2 + 6$$
$$= 8$$
$$b = -8$$

Product of roots, POR

$$c = 2 \times 6$$
$$c = 12$$

(b) $f(x) = x^2 - 8x + 12$

$$f(x) = (4)^2 - 8(4) + 12$$
$$= -4$$

(4, -4)

Solution 17 :

(c) the range of values of x when $f(x)$ is negative,
(d) the maximum value when the graph is reflected in the x-axis.

(c) $f(x) = x^2 - 8x + 12$

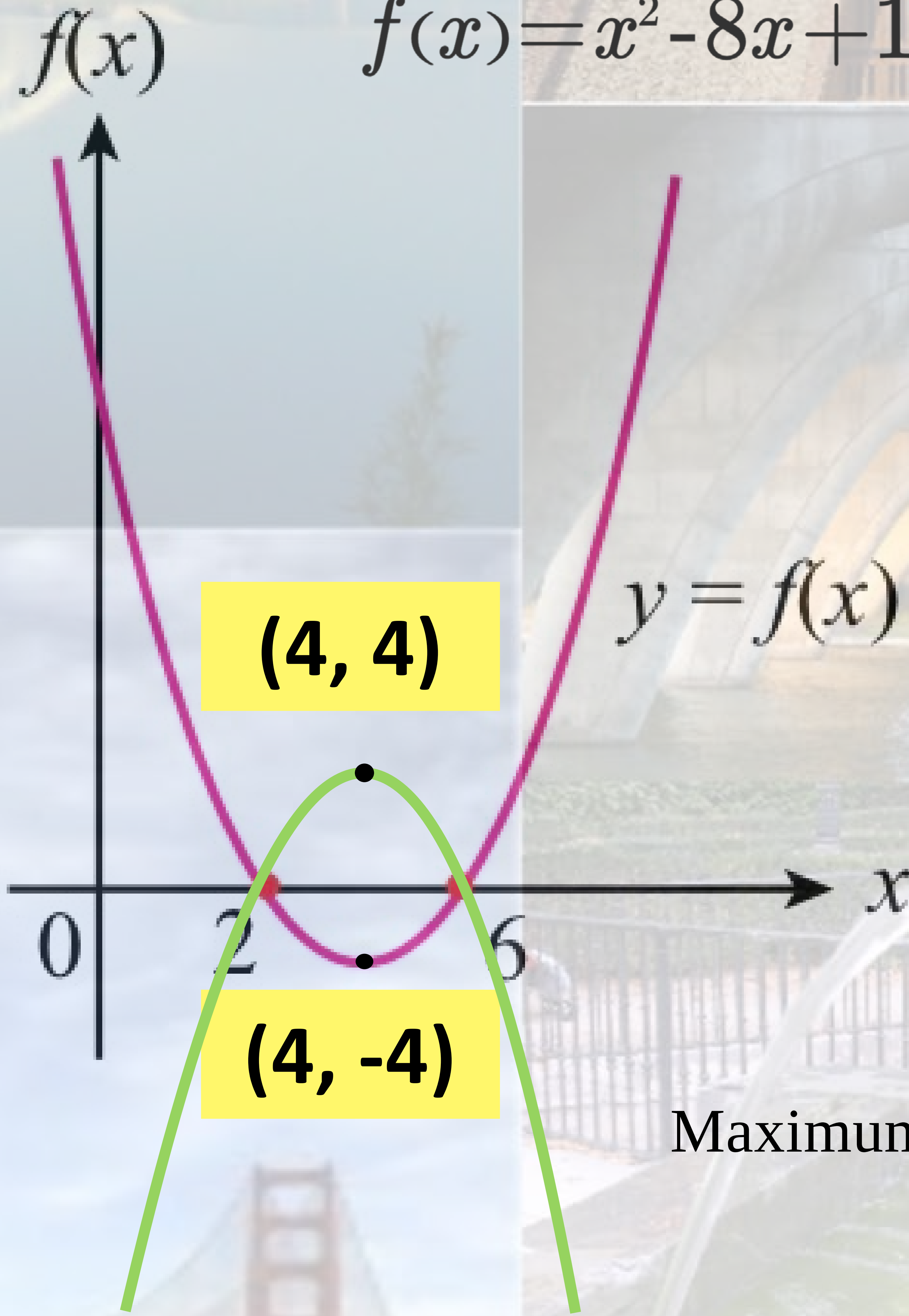
$f(x) < 0$

$x^2 - 8x + 12 < 0$

$(x-2)(x-6) < 0$

$2 < x < 6$

(d) $f(x) = x^2 - 8x + 12$



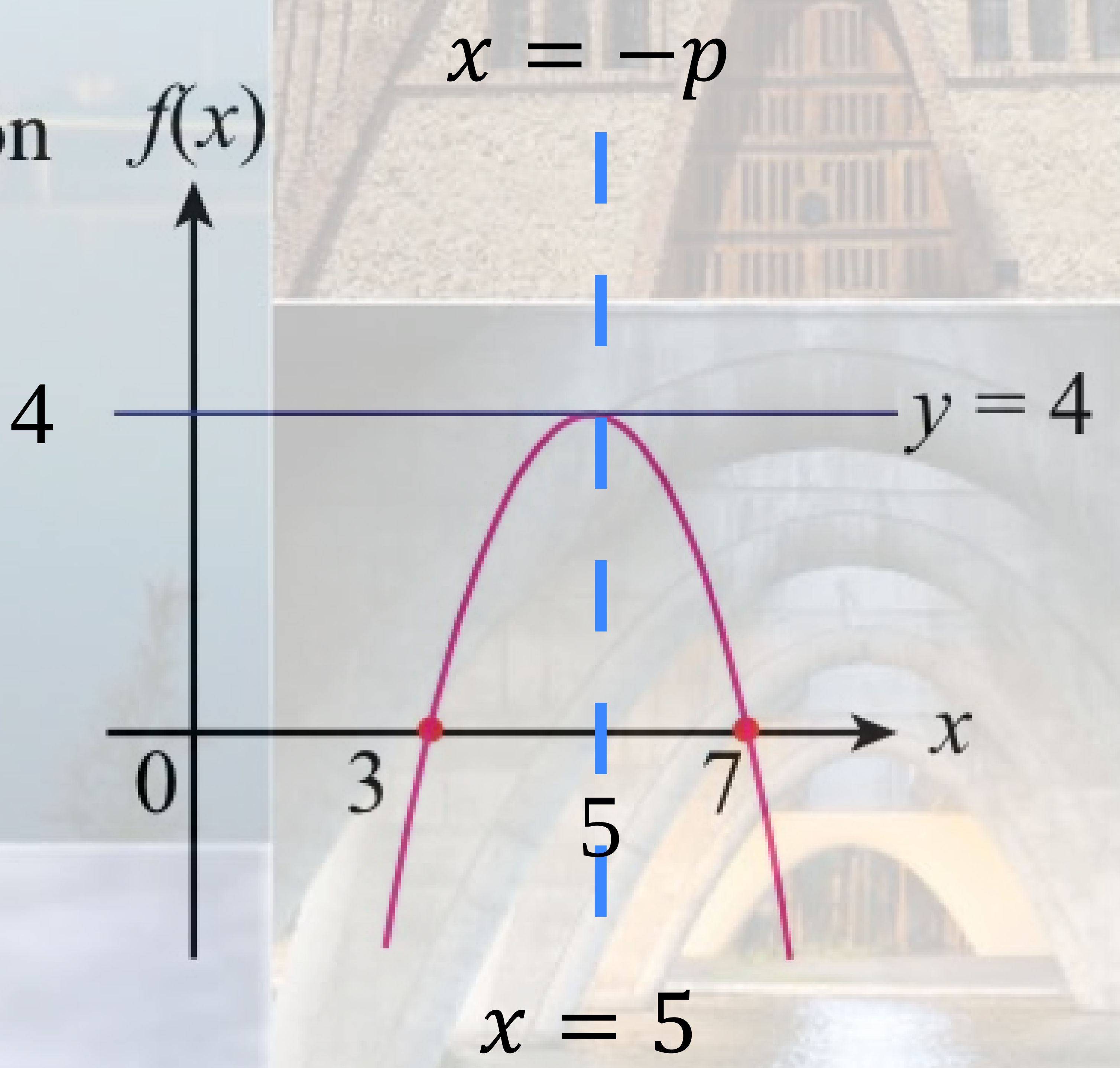
Maximum value = 4

SPM Practice

Example 18

The diagram on the right shows the graph of quadratic function $f(x) = -\frac{1}{3}[(x+p)^2 + q]$. The line $y = 4$ is the tangent to the curve. Find,

(a) the roots for $f(x) = 0$,
(b) the values of p and of q,
(c) the equation of the axis of symmetry of the curve.



Solution 18 :

(a) $x = 3, \; x = 7$

(b) $p = -5$

$q = 4$

(c) $x = 5$

Example 19

SPM Practice

Diagram 1 shows the graph of a quadratic function $f(x) = \frac{p}{x^n} + qx + r$ such that p, q, r, n and u are constants.

Rajah 1 menunjukkan graf bagi fungsi kuadratik $f(x) = \frac{p}{x^n} + qx + r$ dengan keadaan p, q, r, n dan u ialah pemalar.

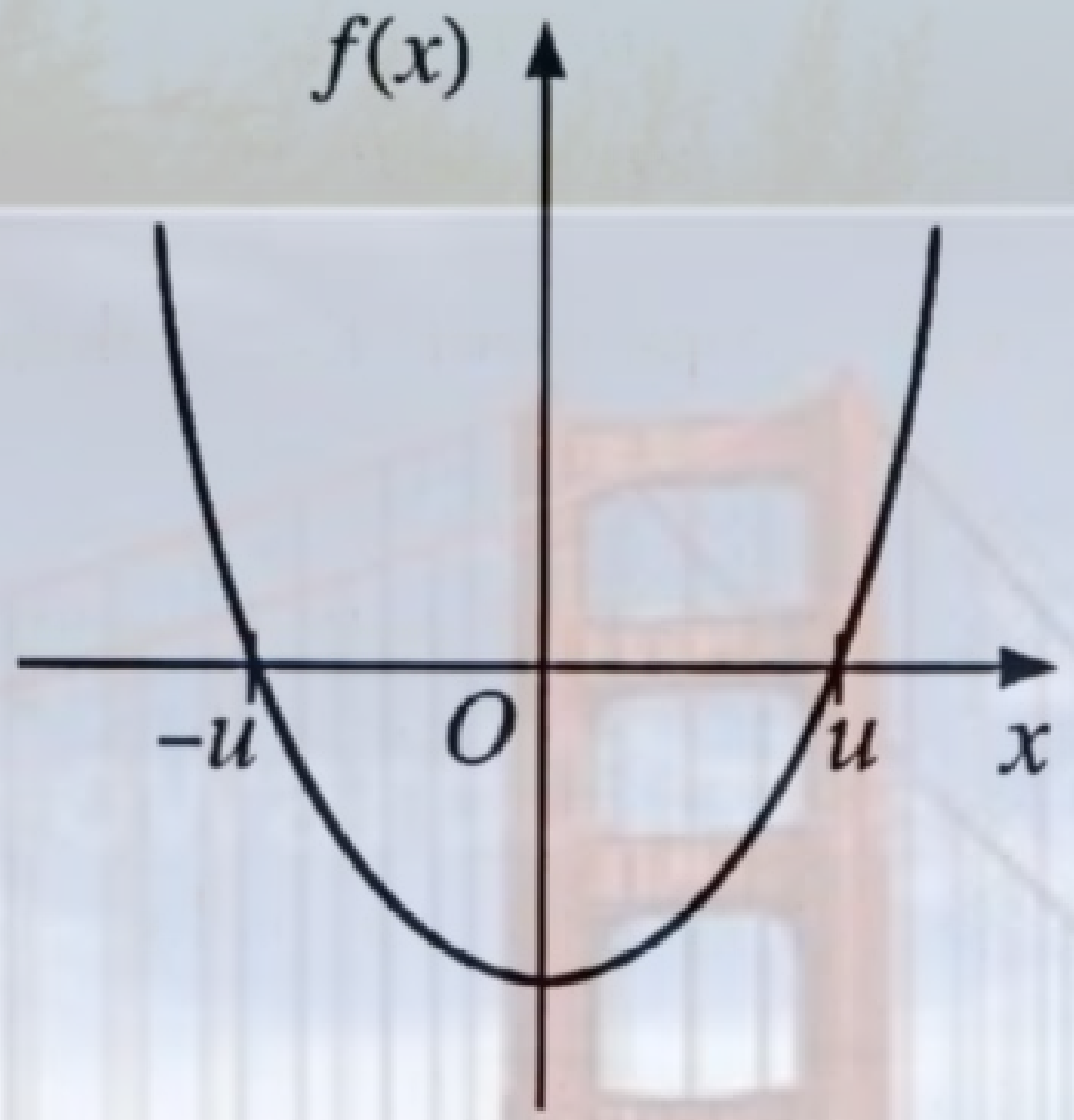


Diagram 1

Rajah 1

(*a*) State the value of n.

Nyatakan nilai n.

(*b*) If $f(x) = 0$ and the product of roots is r, state the value of

Jika $f(x) = 0$ dan hasil darab punca ialah r, nyatakan nilai

(i) q,

(ii) p.

[3 *marks*]

[3 *markah*]

Solution 19(a):

(*a*) State the value of *n*.

Nyatakan nilai n.

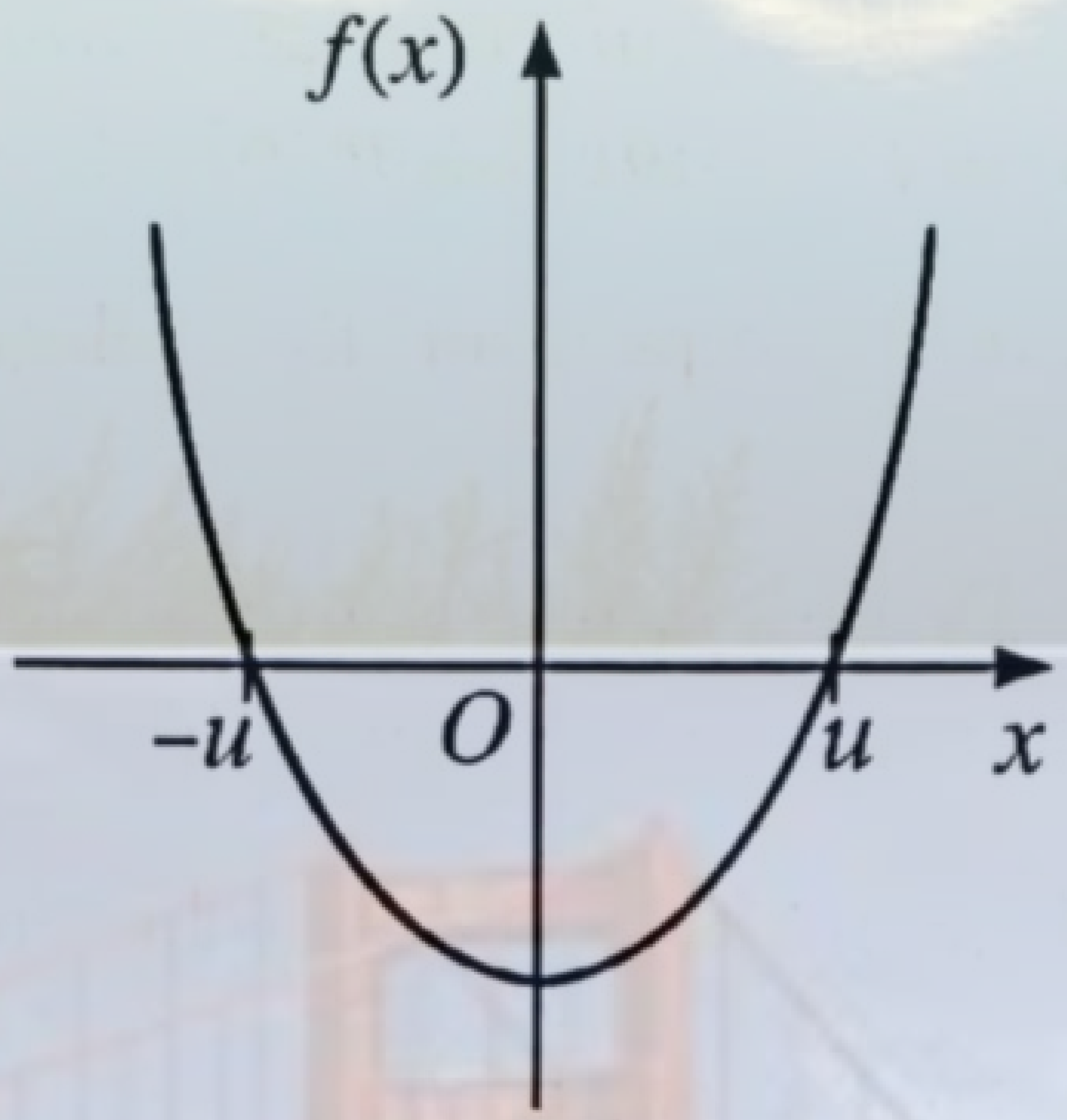


Diagram 1

Rajah 1

$$f(x)=\frac{p}{x^n}+qx+r$$

$$f(x)=px^{-n}+qx+r$$

$$f(x)=px^2+qx+r$$

Compare

$$-n=2$$

$$n=-2$$

Solution 19(b):

(*b*) If $f(x) = 0$ and the product of roots is r, state the value of

Jika $f(x) = 0$ *dan hasil darab punca ialah r, nyatakan nilai*

(i) q,

(ii) p.

$$f(x) = px^2 + qx + r$$

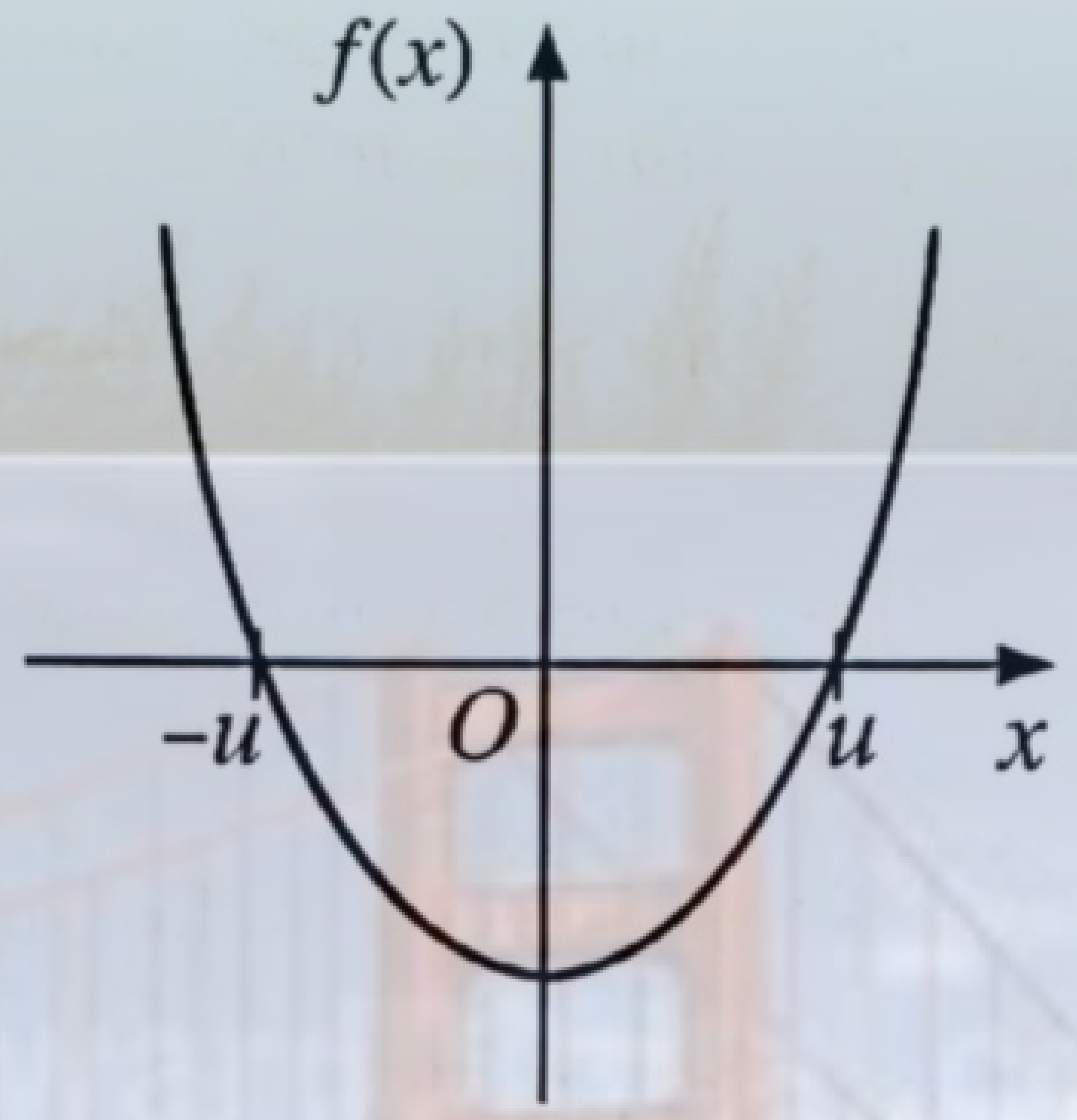


Diagram 1

Rajah 1

$$px^2 + qx + r = 0$$

$$\frac{px^2}{p} + \frac{qx}{p} + \frac{r}{p} = 0$$

$$x^2 + \frac{q}{p}x + \frac{r}{p} = 0$$

$$\text{SOR} = -\frac{q}{p}$$

$$-u + u = \frac{-q}{p}$$

$$q = 0$$

$$\text{POR} = \frac{r}{p}$$

$$r = \frac{r}{p}$$

$$p = \frac{r}{r}$$

$$p = 1$$

Example 20

Diagram 4 shows a graph of a quadratic function $f(x)=(x-p)^2+q$, where p and q are constants. A straight line $y=8$ is a tangent to the curve $y=f(x)$.

Rajah 4 menunjukkan suatu graf fungsi kuadratik $f(x)=(x-p)^2+q$, dengan keadaan p dan q ialah pemalar. Garis lurus $y=8$ ialah tangen kepada lengkung $y=f(x)$.

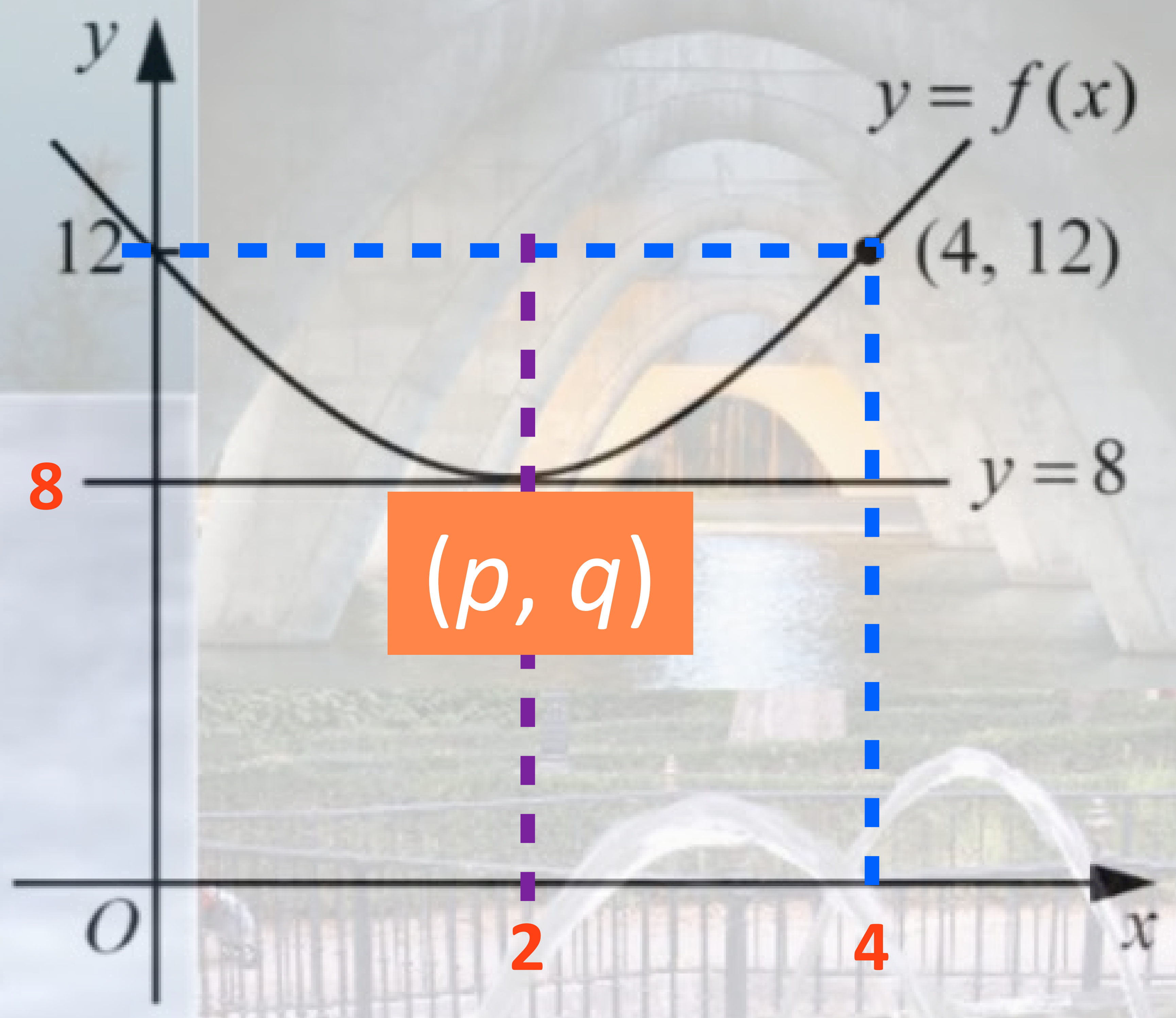


Diagram 4 / *Rajah 4*

State

Nyatakan

(*a*) the value of p,

nilai p,

(*b*) the value of q

nilai q,

(*c*) the equation of the axis of symmetry

persamaan paksi simetri.

[3 *marks*] / [3 *markah*]

Solution 20 :

$$p=2$$

$$q=8$$

$$x=2$$

Example 21

SPM Practice

The graph of the quadratic function $f(x) = x^2 - 3x - m$ intersects the x-axis at $2h$ and $2k$ while the quadratic equation $mx^2 + nx - 1 = 0$ has roots h and k, where m and n are constants and $m > 0$.

Graf bagi fungsi quadratik $f(x) = x^2 - 3x - m$ menyilang paksi-x pada 2h dan 2k manakala persamaan kuadratik $mx^2 + nx - 1 = 0$ mempunyai punca-punca h dan k, dengan keadaan m dan n ialah pemalar dan $m > 0$.

(*a*) Find the value of *m* and of *n*. [3 *marks*]

Cari nilai m dan nilai n. [3 *markah*]

(*b*) (i) Hence, by using the method of completing the square, find the minimum value of $f(x)$.

Seterusnya, dengan menggunakan kaedah penyempurnaan kuasa dua, cari nilai minimum bagi $f(x)$.

(ii) Sketch the graph of $f(x)$.

Lakar graf bagi $f(x)$.

[4 *marks*]
[4 *markah*]

Solution 21(a):

The graph of the quadratic function $f(x) = x^2 - 3x - m$ intersects the x-axis at $2h$ and $2k$ while the quadratic equation $mx^2 + nx - 1 = 0$ has roots h and k, where m and n are constants and $m > 0$.

(*a*) Find the value of m and of n. [3 *marks*]

Cari nilai m dan nilai n. [3 *markah*]

$$f(x) = 0$$

$$x^2 - 3x - m = 0$$

Sum of roots, SOR

$$2h + 2k = 3 \quad ✓$$

$$2(h + k) = 3$$

$$h + k = \frac{3}{2}$$

Product of roots, POR

$$(2h)(2k) = -m \quad ✓$$

$$4hk = -m$$

$$hk = -\frac{m}{4}$$

$$mx^2 + nx - 1 = 0$$

$$x^2 + \frac{n}{m}x - \frac{1}{m} = 0$$

$$h + k = -\frac{n}{m} \quad ✓$$

P1

$$hk = -\frac{1}{m} \quad ✓$$

$$-\frac{m}{4} = -\frac{1}{m} \quad ✓$$

K1

$$m^2 = 4$$

$$m = 2, -2$$

$$m = 2 \quad ✓ \qquad m > 0$$

$$\frac{3}{2} = -\frac{n}{m}$$

$$\frac{3}{2} = -\frac{n}{2}$$

$$n = -3 \quad ✓$$

N1

Solution 21(b)

(b) (i) Hence, by using the method of completing the square, find the minimum value of $f(x)$.

Seterusnya, dengan menggunakan kaedah penyempurnaan kuasa dua, cari nilai minimum bagi $f(x)$.

(ii) Sketch the graph of $f(x)$.

Lakar graf bagi $f(x)$.

[4 *marks*]
[4 *markah*]

$$f(x)=x^2-3x-m$$

$$f(x)=x^2-3x-2$$

Completing the square

$$=x^2-3x+\left(\frac{-3}{2}\right)^2-\left(\frac{-3}{2}\right)^2-2$$ ✓

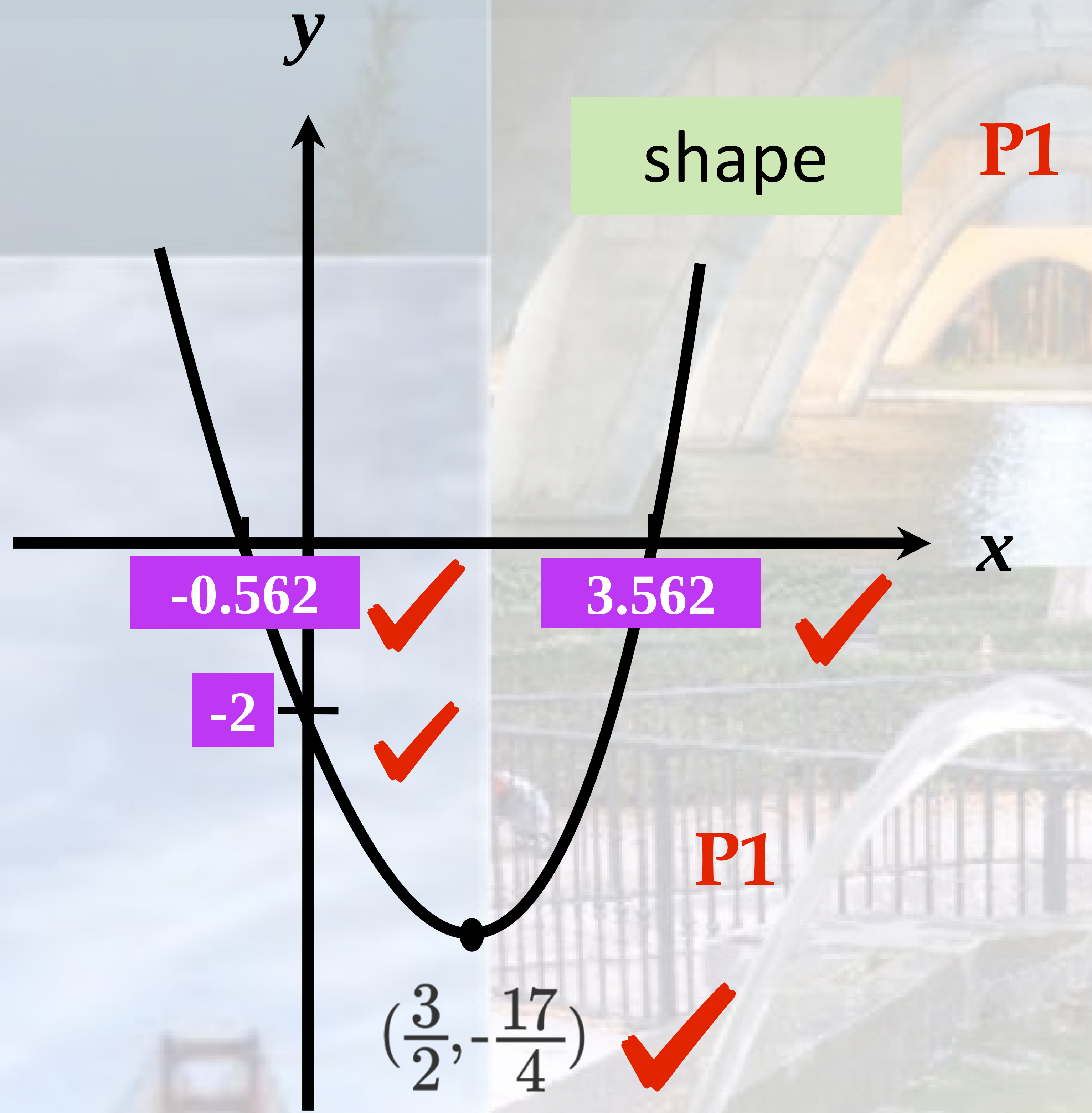
$$=\left(x-\frac{3}{2}\right)^2-\frac{9}{4}-2$$ ✓

K1

Minimum value of $f(x)=-\frac{9}{4}-2$

$$=-\frac{17}{4}$$ ✓

N1



SUMMARY OF QUADRATIC FUNCTIONS